

- 7 A group of scientists is studying how a population of insects grows in the laboratory.
At the start of the study, there are 48 insects.
At the end of the first month, there are 72 insects.
At the end of the second month, there are 108 insects.
The group develops two models, P and Q , to model the population.

The first model, P , is of the form $P_{t+1} = kP_t$, where k is a constant.

P_0 denotes the number of insects at the start of the study.

P_t denotes the number of insects at the end of month t , $t \geq 1$.

Model P fits the data for $t = 0, 1$ and 2 exactly.

(a) Find the value of k . [1]

(b) Use model P to predict the number of insects at the end of month 3. [1]

The second model, Q , is of the form $Q = 48e^{0.4t}$, where t is the time, in months, after the start of the study and Q denotes the number of insects at time t .

(c) (i) By considering the given data on the number of insects when $t = 1$ and $t = 2$, assess whether model Q fits the data for these times. [2]

(ii) Show that model Q implies that the rate of growth of the population is proportional to the population at any time t . [2]

(iii) Show that, for integer values of t with $t > 1$, the population predicted by model Q will always be lower than the population predicted by model P . [3]