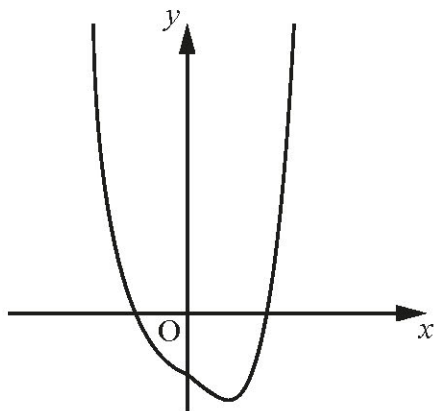


5 The diagram shows the curve with equation $y = x^4 - 2^x$, for values of x close to zero.



(a) In this question you must show detailed reasoning.

Show that the equation $x^4 - 2^x = 0$ has a root which lies between 1 and 2. [2]

(b) Show that the equation $x^4 - 2^x = 0$ can be written in the form $x = 2^{0.25x}$ for positive values of x . [1]

(c) Use the iterative formula $x_{n+1} = 2^{0.25x_n}$ with $x_0 = 0.4$ to determine a root of $x^4 - 2^x = 0$ to 2 decimal places. [2]

(d) The diagram in the Printed Answer Booklet shows the line with equation $y = x$ and the curve with equation $y = 2^{0.25x}$.

Sketch a cobweb or staircase diagram, on the diagram in the Printed Answer Booklet, for the iterative formula $x_{n+1} = 2^{0.25x_n}$ starting with $x_0 = 0.4$. Show at least **two** iterations. [2]

(e) Show that the equation $x^4 - 2^x = 0$ can be written in the form $x = 4 \log_2 x$ for positive values of x . [1]

(f) In this question you must show detailed reasoning.

Show that the iteration $x_{n+1} = 4 \log_2 x_n$, with $x_0 = 1$, will **not** find a root of $x^4 - 2^x = 0$. [2]

5(d)

