14 **Fig. 14.1** shows the curve with equation $y = \frac{1}{1+x^2}$, together with 5 rectangles of equal width.

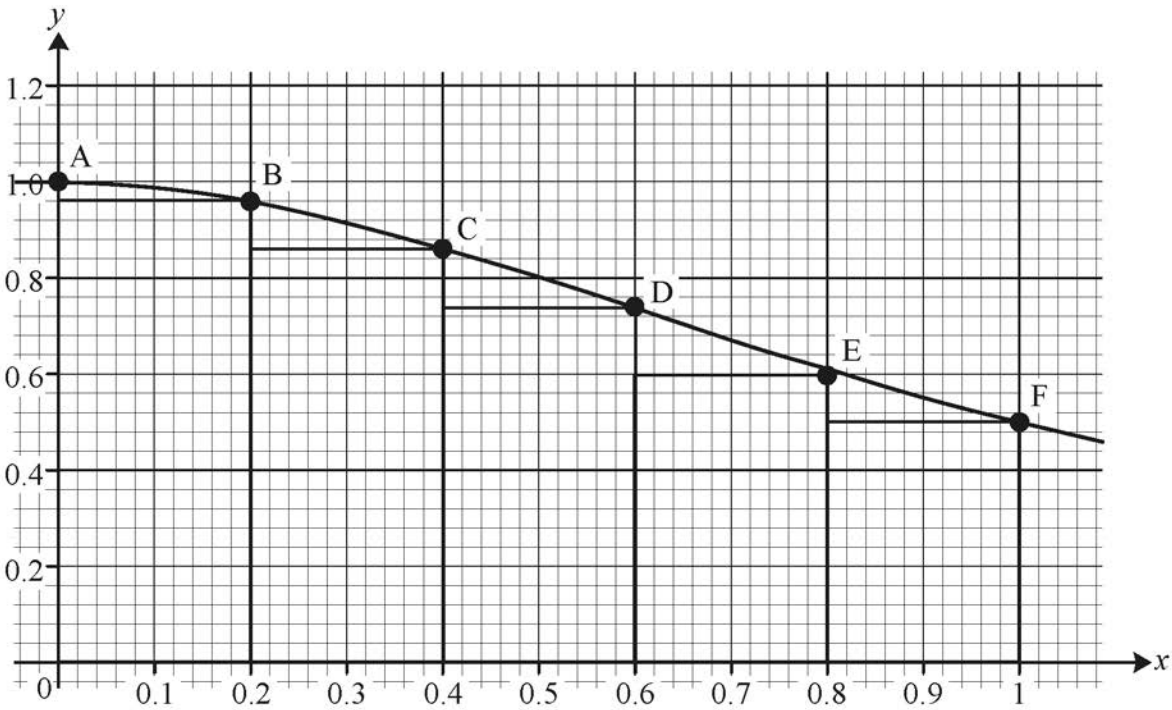

Fig. 14.1

Fig. 14.2 shows the coordinates of the points A, B, C, D, E and F.

Point	A	B	C	D	E	F
x	0	0.2	0.4	0.6	0.8	1
y	1	0.96154	0.86207	0.73529	0.60976	0.5

Fig. 14.2

(a) Use the 5 rectangles shown in **Fig. 14.1** and the information in **Fig. 14.2** to show that a lower bound for $\int_0^1 \frac{1}{1+x^2}\,dx$ is 0.7337, correct to **4** decimal places. **[2]**

(b) Use the 5 rectangles shown in **Fig. 14.1** and the information in **Fig. 14.2** to calculate an upper bound for $\int_0^1 \frac{1}{1+x^2}\,dx$ correct to **4** decimal places. **[2]**

(c) Hence find the length of the interval in which your answers to parts **(a)** and **(b)** indicate the value of $\int_0^1 \frac{1}{1+x^2}\,dx$ lies. **[1]**

Amit uses n rectangles, each of width $\frac{1}{n}$, to calculate upper and lower bounds for $\int_0^1 \frac{1}{1+x^2}\,dx$, using different values of n. His results are shown in **Fig. 14.3**.

n	10	20	40
upper bound	0.80998	0.79779	0.79162
lower bound	0.75998	0.77279	0.77912

Fig. 14.3

(d) Find the length of the smallest interval in which Amit now knows $\int_0^1 \frac{1}{1+x^2}\,dx$ lies. **[2]**

(e) **Without** doing any calculation, explain how Amit could find a smaller interval which contains the value of $\int_0^1 \frac{1}{1+x^2}\,dx$. **[1]**