

12 A student dissolves 0.5 kg of salt in a bucket of water. Water leaks out of a hole in the bucket so the student lets fresh water flow in so that the bucket stays full.

They assume that the salty water remaining in the bucket mixes with the fresh water that flows in, so the concentration of salt is uniform throughout the bucket.

They model the mass M kg of salt remaining after t minutes by $M = ak^t$ where a and k are constants.

(a) Show that the model for M can be rewritten in the form $\log_{10} M = t \log_{10} k + \log_{10} a$. **[1]**

The student measures the concentration of salt in the bucket at certain times to estimate the mass of the salt remaining. The results are shown in the table below.

t minutes	8	13	21	35	50
M kg	0.4	0.3	0.2	0.1	0.05

The student uses this data and plots $y = \log_{10} M$ against $x = t$ using graph drawing software. The software gives $y = -0.0214x - 0.2403$ for the equation of the line of best fit.

(b) (i) Find the values of a and k that follow from the equation of the line. **[3]**

(ii) Interpret the value of k in context. **[1]**

(c) It is known that when $t = 0$ the mass of salt in the bucket is 0.5 kg.

Comment on the accuracy when the model is used to estimate the initial mass of the salt. **[2]**

(d) Use the model to predict the value of t at which $M = 0.01$ kg. **[2]**

(e) Rewrite the model for M in the form $M = ae^{-ht}$ where h is a constant to be determined. **[2]**

(f) Find the rate at which the mass of salt is decreasing when $t = 10$. **[2]**