

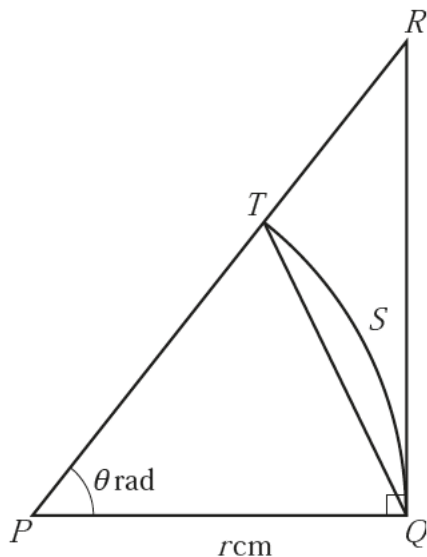
6 The compound angle formulae for $\sin(A+B)$ and $\sin(A-B)$ are

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \text{ and}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B.$$

(a) By letting $C = A + B$ and $D = A - B$, show that

$$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right). \quad [1]$$



The diagram shows a right-angled triangle PQR with $PQ = r$ cm. The angle QPR is θ radians. The diagram also shows the sector $PQST$ of a circle with centre P and radius r cm. The line segment QT is a chord of the sector $PQST$.

(b) By considering the areas of triangle PQT , sector $PQST$ and triangle PQR , show that

$$1 < \frac{\theta}{\sin \theta} < \frac{1}{\cos \theta}. \quad [4]$$

(c) (i) Hence write down a similar inequality interval for the expression $\frac{\sin \theta}{\theta}$. [1]

(ii) Hence state $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$. [1]

- (d)** Using parts **(a)** and **(c)(ii)**, show from first principles that the derivative of $\sin x$ is $\cos x$, where x is measured in radians. **[4]**

A student attempts to use the result regarding the derivative of $\sin x$ to find the derivative of $\cos x$. The student's attempt is shown below.

Let $y = \cos x$, where x is measured in radians.

$$y = \sin\left(\frac{\pi}{2} - x\right)$$

so $\frac{dy}{dx} = \cos\left(\frac{\pi}{2} - x\right)$

but $\sin x \equiv \cos\left(\frac{\pi}{2} - x\right)$

therefore $\frac{dy}{dx} = \sin x$.

- (e)** Identify the error made by the student.

[1]