

5 (a) Use the substitution $u = \cos x$ to show that

$$\int \frac{1 + \cos x}{\sin x} dx = \ln(1 - \cos x) + c. \quad [4]$$

Fig. 1

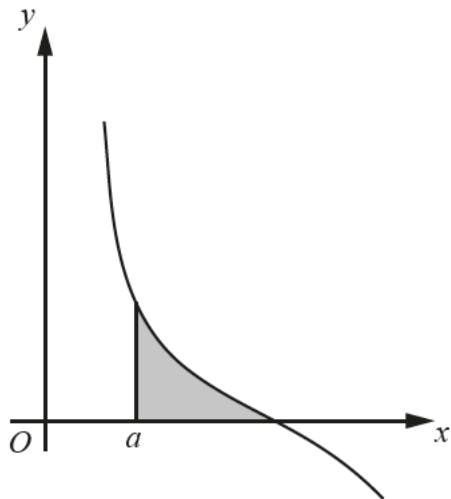


Fig. 1 shows part of the curve $y = \frac{1 + \cos x}{\sin x}$.

The shaded region is bounded by the curve, the x -axis, and the line $x = a$.

You are given that the area of the shaded region is a square units.

(b) Show that the value of a satisfies the equation $a - \cos^{-1}(1 - 2e^{-a}) = 0$. [4]

(c) Show by calculation that the value of a lies between 1.1 and 1.2. [2]

(d) Use the iterative formula

$$a_{n+1} = \cos^{-1}(1 - 2e^{-a_n}),$$

with starting value $a_1 = 1.2$, to find the value of a correct to 2 decimal places. Show the result of each step of the iterative process. [2]

Fig. 2

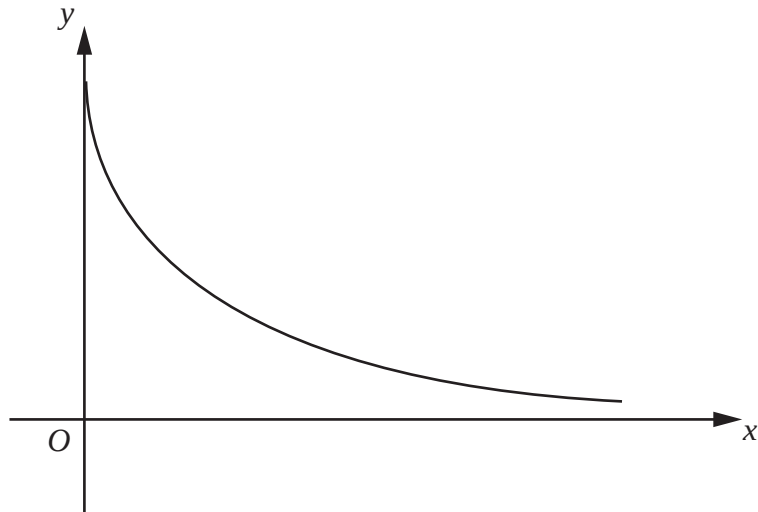


Fig. 2 shows the curve $y = \cos^{-1}(1 - 2e^{-x})$.

- (e)** Explain why the gradient of the curve $y = \cos^{-1}(1 - 2e^{-x})$ at the point where $x = a$, where a is the value found in part **(d)**, lies in the interval $(-1, 0)$. **[2]**