

Question		Answer	Marks	AO	Guidance	
5	(a)	$u = \cos x \Rightarrow du = -\sin x \, dx$ $\int \frac{1 + \cos x}{\sin x} \, dx = -\int \frac{1 + u}{1 - u^2} \, du$ $-\int \frac{1 + u}{(1 - u)(1 + u)} \, du = -\int \frac{1}{1 - u} \, du$ $= -(-\ln(1 - u)) \quad (+c) = \ln(1 - \cos x) + c$	B1 M1* M1dep* A1 [4]	2.1 1.1 3.1a 2.1	B1 for $\frac{du}{dx} = -\sin x$ oe Obtain an integral in terms of u only – must be of the form $\pm \int \frac{\pm 1 \pm u}{\pm 1 \pm u^2} (du)$ Explicit use of $1 - u^2 = (1 + u)(1 - u)$ or $u^2 - 1 = (u + 1)(u - 1)$ and simplify to an integral of the form $\pm \int \frac{1}{\pm 1 \pm u} (du)$ AG www – condone $\ln 1 - \cos x + c$ Allow A1 for $-\int \frac{1}{1 - u} \, du = \ln(1 - u) = \ln(1 - \cos x) + c$ (must see $\ln(1 - u)$ as AG) However, $\int \frac{1}{u - 1} \, du = \ln \cos x - 1 = \ln(1 - \cos x) + c$ is A0 without further correct working or correct explanation/justification Note that $\int \frac{1}{u - 1} \, du = \ln(\cos x - 1) = \ln(1 - \cos x) + c$ is A0 regardless of any further explanation/justification	Allow du missing throughout but M0 if implying $du = dx$ $+c$ must be present in the final given answer e.g. as a minimum $\ln \cos x - 1$ $= \ln (-1)(1 - \cos x)$ $= \ln(1 - \cos x) + c$

Question		Answer	Marks	AO	Guidance	
5	(b)	Curve intersects the x-axis at π $\left[\ln(1 - \cos x)\right]_a^\pi = a \Rightarrow \ln(1 - \cos \pi) - \ln(1 - \cos a) = a$ $\ln\left(\frac{2}{1 - \cos a}\right) = a \Rightarrow \frac{2}{1 - \cos a} = e^a$ $1 - \cos a = 2e^{-a} \Rightarrow \cos a = 1 - 2e^{-a}$ so $a = \cos^{-1}(1 - 2e^{-a})$ and therefore $a - \cos^{-1}(1 - 2e^{-a}) = 0$	B1 M1* M1dep* A1 [4]	3.1a 1.1 1.1 2.1	Possibly implied by later working – condone (use of) 180 (in degrees) or $\cos^{-1}(-1)$ (so do not need to see this evaluated to π) Correct use of their upper limit and correct lower limit to form an equation in a only so must be of the form $\ln k - \ln(1 - \cos a) = a$ following through their value of $k > 0$ Correct use of log laws and correctly remove natural log to form an equation in e^a from $\ln k - \ln(1 - \cos a) = a$ following through their $k > 0$ and $k \neq 1$ AG – sufficient working must be shown and must be = 0	e.g. as a limit on an integral Setting equal to a^2 rather than a is M0 allow arccos for $\cos^{-1}$
5	(c)	Let $f(a) = a - \cos^{-1}(1 - 2e^{-a})$ $f(1.1) = -0.129... < 0$, $f(1.2) = 0.038... > 0$ Change of sign indicates that a lies between 1.1 and 1.2	M1 A1 [2]	1.1 2.4	Obtain correct value for either $f(1.1)$ or $f(1.2)$ to at least 1 sf rot Both correct values to at least 1 sf rot, ‘change of sign’ either shown or stated + conclusion (minimum e.g. ‘root’)	Allow, for example, $-0.1 < 0 < 0.03$ therefore $1.1 < a < 1.2$’ for A1

Question			Answer	Marks	AO	Guidance	
5	(d)		$a_2 = 1.16(188...), a_3 = 1.18(725...), a_4 = 1.17(02...)$	B1	1.1	At least the first three values of a_i stated correctly to at least 2 decimal places rot e.g. $a_3 = 1.18$ or $a_3 = 1.19$	Ignore values after the first three
			$a = 1.18$	B1	1.1	cao (must be to exactly 2 decimal places) – final answer must be 1.18 or $a = 1.18$ and not a term of a sequence e.g. $a_9 = 1.18$	Not dependent on the previous B1. A correct answer without the correct first three terms is B0 B1
				[2]			

Question			Answer	Marks	AO	Guidance	
5	(e)		The diagram shows that the gradient of all points on the curve $y = \cos^{-1}(1 - 2e^{-x})$ are negative and therefore the gradient at a lies in the interval $(-\infty, 0)$	B1	2.4	Correct explanation that the graph or diagram or curve (in Fig. 2) implies that the gradient (either at a or implying all points) is negative .	Must mention ‘the graph’ oe for this mark but mention of Fig. 1 is B0
			As the iterative formula in part (d) converged to the value of a this implies that $\left \frac{d}{dx}(\cos^{-1}(1 - 2e^{-x})) \right _{x=a} < 1$ and so the gradient of the curve at the point $x = a$ lies in the interval $(-1, 0)$.	B1	2.4	Correct justification that as the iterative formula $(a_{n+1} = F(a_n))$ converged to a $\Rightarrow F'(a) < 1$. As a minimum must state/mention ‘convergence’ and EITHER that the gradient at a is between -1 and 1 OR that the gradient at a is between -1 and 0 but only following a correct explanation that the graph (oe) implies a negative gradient	Not $\Rightarrow F'(a_n) < 1$. For both marks to be awarded the interval of $(-1, 0)$ must be mentioned
				[2]		B2 for ‘graph shows negative gradient, and there is convergence, therefore the gradient at a lies in the interval $(-1, 0)$ ’	Implying that a is between -1 and 1 and not the gradient is B0 for the second B mark