

Question	Answer	Mark	AO	Guidance
8	DR $x \ln(3 - 2x) + \int \frac{2x}{3 - 2x} dx$ $= x \ln(3 - 2x) + \int \left(-1 + \frac{3}{3 - 2x}\right) dx$ $= x \ln(3 - 2x) - x - \frac{3}{2} \ln(3 - 2x) [+c]$ $\left(\frac{1}{2} \ln 2 - \frac{1}{2} - \frac{3}{2} \ln 2\right) - \left(-\frac{3}{2} \ln 3\right) \text{ oe}$ $= \frac{3}{2} \ln 3 - \ln 2 - \frac{1}{2} \quad \text{or} \quad \ln \frac{3\sqrt{3}}{2} - \frac{1}{2} \text{ oe}$	M1* M1** dep* A1 M1 dep** A1	3.1a 3.1a 1.1 2.1 1.1	Condone missing 'dx' or 'du' throughout all methods. Attempt integration by parts – must have correct parts, possibly with $\int x \times (-2) \times \frac{1}{3-2x} dx$ not yet simplified. Allow a sign error or missing (-)2. No limits needed for M1M1A1 (first 3 marks) Attempt partial fractions; allow one sign error Attempt substitute correct limits $0, \frac{1}{2}$ and subtract (allow one slip). FT their integral. Dependent on both previous M marks. For any correct exact form, including $\left(\frac{1}{2} \ln 2 - \frac{1}{2} - \frac{3}{2} \ln 2\right) - \left(-\frac{3}{2} \ln 3\right)$, need not be simplified. ISW Answer 0.45477... scores A0 unless exact answer also seen
	Alternative method using substitution following correct setup of integration by parts $x \ln(3 - 2x) + \int \frac{2x}{3 - 2x} dx$ $= x \ln(3 - 2x) + \int \frac{3 - u}{u} \left(-\frac{1}{2}\right) du$ $= \frac{3 - u}{2} \ln(u) - \frac{1}{2} [3 \ln u - u]$ $= \frac{1}{2} \ln 2 - \frac{1}{2} (3 \ln 2 - 2) + \frac{1}{2} (3 \ln 3 - 3)$ $= \frac{3}{2} \ln 3 - \ln 2 - \frac{1}{2} \quad \text{or} \quad \ln \frac{3\sqrt{3}}{2} - \frac{1}{2} \text{ oe}$	M1* M1** dep* A1 M1 dep** A1		If candidates apply integration by parts and then use substitution, apply this method. (NB candidates can switch from this back to the main method if they revert correctly from u to x so look for limits used correctly in either x or u as appropriate). Attempt integration by parts – must have correct parts, possibly with $\int x \times (-2) \times \frac{1}{3-2x} dx$ not yet simplified. Allow a sign error or missing '2'. No limits needed for M1M1A1 (first 3 marks) Using e.g. $u = 3 - 2x$, allow one sign error. Ignore any additional constant term e.g. $+c$ or $+k$ for any k. Attempt to substitute transformed limits 3,2 and subtract (condone swapped limits). FT their integral. Dep on both previous M marks. For any correct exact form, including $\left(\frac{1}{2} \ln 2 - \frac{1}{2} - \frac{3}{2} \ln 2\right) - \left(-\frac{3}{2} \ln 3\right)$, need not be simplified. ISW Answer 0.45477... scores A0 unless exact answer also seen

Alternative method for Q8 applying substitution first

$$-\frac{1}{2} \int_3^2 \ln u \, du$$

$$= -\frac{1}{2} \left[u \ln u - \int u \frac{1}{u} du \right]$$

$$= -\frac{1}{2} [u \ln u - u]_3^2$$

$$= -\ln 2 + 1 + \frac{3}{2} \ln 3 - \frac{3}{2} \text{ oe}$$

M1*

A1

**M1
dep***

A1

A1

If candidates start by applying a substitution, then apply this method.

Attempting to integrate by substitution using e.g. $u = 3 - 2x$ (no limits required for this mark). Allow a sign error or missing $-\frac{1}{2}$.

Fully correct integral with transformed limits (may be seen later, condone swapped limits).

Use of integration by parts. This mark is not implied by later correct working – must see this form or e.g. $\ln u, \frac{1}{u}, u$ and 1 as setup for integration by parts. Ignore limits for this mark. Condone a sign error or missing $-\frac{1}{2}$.

oe but must be correct

For any correct form (as above). ISW

NB candidates who do not show evidence of integration by parts used anywhere can score max. 2/5 under this method.

Answer 0.45477... scores A0 unless exact answer also seen

Alternative method for Q8 assuming formula for $\int \ln x \, dx$ (without substitution) (Only 2 marks, since this Q is DR and required integration by parts)

$$-\frac{1}{2} [(3 - 2x) \ln(3 - 2x) - (3 - 2x)]_0^{\frac{1}{2}}$$

$$= \frac{1}{2} \ln 2 - \frac{1}{2} - \frac{3}{2} \ln 2 + \frac{3}{2} \ln 3 \text{ oe}$$

M1

A1

If no evidence of integration by parts, or of substitution, then apply this method.

Allow $k[(3 - 2x) \ln(3 - 2x) - (3 - 2x)]_0^{\frac{1}{2}}$

ISW

[5]

Answer 0.45477... scores A0 unless exact answer also seen