

For a cone with base radius r, height h and slant height l, the following formulae are given.

Curved surface area, $S = \pi r l$

Volume,
$$V = \frac{1}{3}\pi r^2 h$$

A container is to be designed in the shape of an inverted cone with no lid. The base radius is r m and the volume is V m³.

The area of the material to be used for the cone is $4\pi \,\mathrm{m}^2$.

(a) Show that
$$V = \frac{1}{3}\pi\sqrt{16r^2 - r^6}$$
. [4]

(b) In this question you must show detailed reasoning.

It is given that V has a maximum value for a certain value of r.

Find the maximum value of V, giving your answer correct to 3 significant figures.

[5]