

Question			Answer	Marks	AO	Guidance	
11	(a)		a and b are positive integers	B1	2.3	Correct condition Accept use of $\mathbb{Z}^+$, or $\mathbb{Z}$ with $b \neq 0$, when describing a and b	Allow ‘ a and b are integers’ only if also stated that $b \neq 0$
			a and b have no common factors (other than 1)	B1	2.3	Correct condition (‘not 1’ is not required) a and b are coprime The HCF of a and b is 1	Condone $\frac{a}{b}$ is fully simplified (or in its simplest form) Could be implied by ‘Euclid’s infinite descent’ in (c)
				[2]			
11	(b)		b^2 is an integer, and $a^2 = 5b^2$ hence a^2 is a multiple of 5	B1*	2.2a	Identify that a^2 is a multiple of 5	eg 5 is a factor of a^2 eg $\frac{a^2}{5}$ is an integer
			a square number must have pairs of the same factor, so a^2 must be a multiple of 5^2 hence a is a multiple of 5	B1dep*	2.4	Explain why a must also be a multiple of 5 Could refer to prime factors or factors of square numbers	eg $\frac{a \times a}{5}$ is an integer, so a must be a multiple of 5 Not just a^2 is a multiple of 5 hence a is a multiple of 5 as well, as this is given in the question
				[2]		See appendix for exemplars	

Question			Answer	Marks	AO	Guidance	
11	(c)		$(5k)^2 = 5b^2$ $25k^2 = 5b^2$	M1	2.1	Correctly use $a = 5k$ to obtain an equation with no brackets	Condone statement about b, with no reference to b^2 first Or eg $\frac{a}{b} = \frac{5k}{5m}$ hence not in simplest form (as alternative evidence for sharing a common factor)
			$b^2 = 5k^2$ hence b^2 is a multiple of 5, so b is a multiple of 5	A1	2.4	Simplify and state condition on b	
			This means that a and b share a common factor of 5, which contradicts the original assumption that a and b had no common factors.	A1	2.2a	Conclude appropriately, dep on (2 nd) B1 in part (a) for ‘no common factors’ oe If B0 for ‘no common factors’ in part (a) then this mark is A0 , as no assumption to contradict	
			Hence $\sqrt{5}$ cannot be written as $\frac{a}{b}$ so must be irrational .	[3]		Need reference to ‘common factor’ (or simplest form), ‘contradiction’ and ‘irrational’ in conclusion	

Exemplar responses for Q11(b) for the B1dep*

Response	Mark	Comment
The only way for a^2 to be a multiple of 5 is if a is a multiple of 5 or $\sqrt{5}$. Since a is defined as in integer, it cannot be a multiple of $\sqrt{5}$ so it must be a multiple of 5.	B1	Correct reasoning about the factors of a and a^2
The square root of any multiple of 5 must be a multiple of 5 so $\sqrt{a^2}$ is a multiple of 5	B0	No reason given
As a^2 is a multiple of 5, a must also be a multiple of 5 as it is an integer so can't have a factor of $\sqrt{5}$.	B0	Does not explain why there has to be a factor of $\sqrt{5}$.
5 is prime, so 5 can only be factor of a^2 if it is also a factor of a . Hence a is a multiple of 5	B1	Correctly considers prime factorisation
5 is prime so no other numbers can produce it, therefore must come from product of a and itself to appear in a^2 .	B1	Uses the fact that 5 is prime
Since 5 is not a square number, a must be a multiple of 5, so that a^2 is a multiple of 25 then and therefore a multiple of 5.	B1	Considers repeated factors of square numbers
The factors of a^2 are the same as the factors of a . Therefore if 5 is a factor of a^2 , 5 must also be a factor of a , making a a multiple of 5.	B0	First statement incorrect.
If a^2 is a multiple of 5 a will also be a multiple of 5 because they both have a common factor of 5	B0	No explanation as to why they have a common factor of 5.
a is a multiple of 5 since any prime factor of a^2 is also a factor of a	B1	Considers prime factors of square numbers
5 is prime and cannot be split up so must also be a factor of a	B0	Does not relate to a^2
a^2 must have units digit 0 or 5 but this cannot happen unless a is a multiple of 5	B1	Justifies by considering all possible units digits of multiples of 5
If a is an integer its square will have the same prime factors as a , so if a^2 has a prime factor of 5 so does a	B1	Considers prime factors of square numbers
a^2 is a multiple of 5 so some factors of a must multiply to 5, but 5 is prime so the factor in a must be 5	B1	Considers prime factors of a^2 and a
a is a multiple of 5 since only a multiple of 5 multiplied by itself gives a multiple of 5	B1	Uses properties of multiples of 5