

Question	Scheme	Marks	AOs
7	Any 2 of $x = \pm(4-3)$ or $y = \pm(-3-5)$ or $z = \pm(-1-2)$ for $\pm\overline{BC}$	M1	3.1a
	$(\overline{AB} =)\sqrt{4^2 + 3^2 + 1^2}$ or $(\overline{AC} =)\sqrt{3^2 + 5^2 + 2^2}$ or $(\overline{BC} =)\sqrt{1^2 + 8^2 + 3^2}$ NB $(\overline{AB} = \sqrt{26}, \overline{AC} = \sqrt{38}, \overline{BC} = \sqrt{74})$	M1	1.1b
	$74 = 26 + 38 - 2\sqrt{26}\sqrt{38}\cos\theta$ or e.g. $\cos BAC = \frac{38 + 26 - 74}{2\sqrt{38}\sqrt{26}}$ or e.g. $\cos^{-1}\left(\frac{74 - 38 - 26}{-2\sqrt{38}\sqrt{26}}\right)$	ddM1	2.1
	$= 99.2^\circ$	A1	2.2a
		(4)	

(4 marks)

Note that they are not asked for $\overline{BC}$ or $\overline{CB}$ in this question. As such, for the main scheme, all they need are the magnitudes of the components of $\overline{BC}$ or $\overline{CB}$ to find $|\overline{BC}|$ or $|\overline{CB}|$ so you can ignore if $\overline{BC}$ or $\overline{CB}$ is correct or not and full marks can be awarded even if there are sign errors in their $\overline{BC}$ or $\overline{CB}$ if they write it as a vector.

So allow full marks for use of $\pm\overline{BC} = \pm\mathbf{i} \pm 8\mathbf{j} \pm 3\mathbf{k}$ or as a column vector.

Remember to check Figure 1 for lengths/vectors which may imply the method marks.

M1: Attempts to find at least 2 of the components of $\overline{BC}$ or $\overline{CB}$ correctly.

It is for any 2 of $x = \pm(4-3)$ or $y = \pm(-3-5)$ or $z = \pm(-1-2)$.

May be implied by correct values which may be seen as a vector.

$\pm(4-3)\mathbf{i} \pm (-3-5)\mathbf{j} \pm (-1-2)\mathbf{k}$ or as a column vector.

Note that e.g. $\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}$ would score this mark and also gives access to the rest of the marks.

M1: Attempts to find the length or the square of the length **of at least one** side of the triangle

using Pythagoras correctly e.g. $(|\overline{AB}| =)\sqrt{4^2 + 3^2 + 1^2}$ or $(|\overline{AB}|^2 =)4^2 + 3^2 + 1^2$

Or $(|\overline{AC}| =)\sqrt{3^2 + 5^2 + 2^2}$ or $(|\overline{AC}|^2 =)3^2 + 5^2 + 2^2$ or $(|\overline{BC}| =)\sqrt{1^2 + 8^2 + 3^2}$ or

$(|\overline{BC}|^2 =)1^2 + 8^2 + 3^2$

Condone incorrect or no labelling of sides/vectors and just look for a correct calculation or value.

Follow through their components of $\overline{BC}$ or $\overline{CB}$ however obtained.

ddM1: Attempts to use the cosine rule with their values correctly placed for angle BAC .

Allow use of e.g. θ (or any other letter) for angle BAC .

Condone slips in substitution or rearrangement if a correct cosine rule formula is seen but the values should be correctly placed e.g. condone the omission of the “2” if a correct formula is quoted. **Depends on both previous method marks.**

May be implied by the correct value of $\cos BAC$ which is $-0.159\dots$ **but not** $+0.159\dots$ unless the correct cosine rule is seen first and **not** just the correct value of angle BAC

A1: Awrt 99.2° only with or without a degrees symbol. Do not isw if e.g. they subsequently find e.g. $360^\circ - 99.2^\circ$.
 Give BOD for cases where use of the cosine rule leads to an angle of $80.8\dots$ which subsequently becomes 99.2 unless it clearly follows an incorrect cosine rule formula.

Alternative using the scalar product (not on this spec):

	$\pm \overrightarrow{AB} \cdot \overrightarrow{AC} = \pm(12 - 15 - 2) = \pm 5$	M1	3.1a
	$ \overrightarrow{AB} = \sqrt{4^2 + 3^2 + 1^2}$ or $ \overrightarrow{AC} = \sqrt{3^2 + 5^2 + 2^2}$ $ \overrightarrow{AB} = \sqrt{26}$, $ \overrightarrow{AC} = \sqrt{38}$	M1	1.1b
	$\sqrt{38}\sqrt{26} \cos \theta = \pm 5$ or e.g. $\cos \theta = \frac{\pm 5}{\sqrt{38}\sqrt{26}}$ or e.g. $BAC = \cos^{-1}\left(\frac{\pm 5}{\sqrt{38}\sqrt{26}}\right)$	ddM1	2.1
	99.2°	A1	2.2a
		(4)	

Notes

M1: Attempts $\pm \overrightarrow{AB} \cdot \overrightarrow{AC}$
 May be implied by sight of ± 5 if it is clear they are attempting the scalar product or by a correct calculation e.g. $\pm(4 \times 3 + 5 \times (-3) + 2 \times (-1))$

M1: Attempts to find the length or the square of the length **for at least one** of AB or AC using Pythagoras correctly e.g. $(|\overrightarrow{AB}| =) \sqrt{4^2 + 3^2 + 1^2}$ or $(|\overrightarrow{AB}|^2 =) 4^2 + 3^2 + 1^2$
 or $(|\overrightarrow{AC}| =) \sqrt{3^2 + 5^2 + 2^2}$ or $(|\overrightarrow{AC}|^2 =) 3^2 + 5^2 + 2^2$
 Condone incorrect or no labelling of sides/vectors and just look for a correct calculation.

ddM1: Attempts to use the scalar product with the values correctly placed.
 Allow use of e.g. θ (or any other letter) for angle BAC .
 May be implied by $\cos BAC = \pm 0.15907\dots$ but **not** just the correct value of angle BAC
Depends on both previous method marks.

A1: Awrt 99.2° only with or without a degrees symbol.
 Do not isw if e.g. they subsequently find $360^\circ - 99.2^\circ$

Alternative using the vector product (not on this spec):

	$\overrightarrow{AB} \times \overrightarrow{AC} = -\mathbf{i} - 11\mathbf{j} + 29\mathbf{k}$	M1	3.1a
	$ \overrightarrow{AB} = \sqrt{4^2 + 3^2 + 1^2}$ or $ \overrightarrow{AC} = \sqrt{3^2 + 5^2 + 2^2}$ $ \overrightarrow{AB} = \sqrt{26}$, $ \overrightarrow{AC} = \sqrt{38}$	M1	1.1b
	$\sin \alpha = \frac{\sqrt{1^2 + 11^2 + 29^2}}{\sqrt{38}\sqrt{26}}$	ddM1	2.1
	$(BAC =) 180 - \sin^{-1}\left(\frac{\sqrt{963}}{\sqrt{38}\sqrt{26}}\right) = 99.2^\circ$	A1	2.2a
		(4)	

Notes

M1: Attempts to find at least 2 components of $\overrightarrow{AB} \times \overrightarrow{AC}$

It is for any 2 of $x = \pm(-6+5)$ or $y = \pm(8+3)$ or $z = \pm(20+9)$.

May be implied by correct values which may be seen as a vector e.g.

$$\pm(-6+5)\mathbf{i} \pm(8+3)\mathbf{j} \pm(20+9)\mathbf{k} \text{ or as a column vector e.g. } \begin{pmatrix} \pm(-6+5) \\ \pm(8+3) \\ \pm(20+9) \end{pmatrix}$$

M1: Attempts to find the length or the square of the length AB or AC using Pythagoras

correctly e.g. $(|\overrightarrow{AB}| = \sqrt{4^2 + 3^2 + 1^2})$ or $(|\overrightarrow{AB}|^2 = 4^2 + 3^2 + 1^2)$

or $(|\overrightarrow{AC}| = \sqrt{3^2 + 5^2 + 2^2})$ or $(|\overrightarrow{AC}|^2 = 3^2 + 5^2 + 2^2)$

Condone incorrect or no labelling of sides/vectors and just look for a correct calculation.

ddM1: Attempts to use the vector product with the values correctly placed.

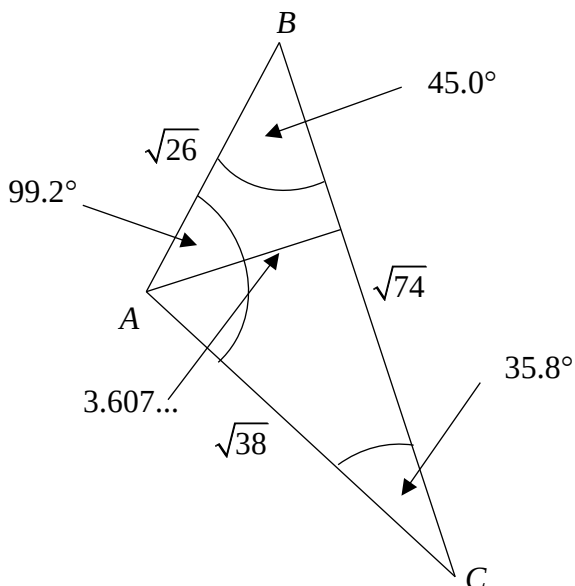
May be implied by the correct value of $\sin BAC$ which is 0.98727... but **not** just the correct value of angle BAC

Depends on both previous method marks.

A1: Awrt 99.2° only with or without a degrees symbol.

Note that they need to subtract their value from 180° to reach the correct answer with this method.

Q7 Diagram/values for reference:



Note that there may be more convoluted methods

e.g.

$$\pm \overrightarrow{BC} = \pm \mathbf{i} \pm 8\mathbf{j} \pm 3\mathbf{k} \text{ or e.g. } \begin{pmatrix} \pm 1 \\ \pm 8 \\ \pm 3 \end{pmatrix} \Rightarrow |\overrightarrow{BC}| = \sqrt{74}$$

$$38 = 26 + 74 - 2\sqrt{26}\sqrt{74} \cos B \Rightarrow B = 45.02977...^\circ$$

Perpendicular distance A to BC is $\sqrt{26} \sin 45.02977...^\circ = 3.60742...$

$$\text{So } \frac{1}{2} \sqrt{74} \times 3.607424... = \frac{1}{2} \sqrt{26} \sqrt{38} \sin \alpha \Rightarrow \alpha = 80.8470...^\circ$$

But A is obtuse so $A = 180^\circ - 80.8470...^\circ = 99.2^\circ$

In such cases the main MS can be applied for the first 2 method marks and then the ddM1 mark would require a complete and correct method to find e.g. angle B and then the perpendicular height followed by an equation using the areas involving angle B to at least find an acute value for angle A and then $A1$ for awrt 99.2

They may also unnecessarily find angle B and angle C then subtract from 180°

The first 2 method marks can be applied as in the main scheme but then the 3rd method would need to be for a complete method for angle A .