15. (a) Find the values of the constants A, B and C for which

$$\frac{u^2}{u^2-1} \equiv A + \frac{B}{u+1} + \frac{C}{u-1}$$

(3)

(b) Use the substitution $u = \sqrt{1+e^{3x}}$ to show that

$$\int_{\frac{1}{3}\ln 3}^{\frac{1}{3}\ln 8} \sqrt{1+e^{3x}}\,dx = k\int_{\alpha}^{\beta} \frac{u^2}{u^2-1}\,du$$

where k, α and β are constants to be found.

(5)

(c) Using the answers from parts (a) and (b) and algebraic integration, show that

$$\int_{\frac{1}{3}\ln 3}^{\frac{1}{3}\ln 8} \sqrt{1+e^{3x}}\,dx = p + q\ln r$$

where p, q and r are rational numbers to be found.

(4)