13. (a) Express $\sin x + \frac{1}{2}\cos x$ in the form $R\sin(x+\alpha)$ where R and α are constants,

$R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, of the water in a fish tank during a particular day is modelled by the equation

$$\theta = 26 + \sin\left(\frac{\pi t}{3} - 8\right) + \frac{1}{2}\cos\left(\frac{\pi t}{3} - 8\right) \qquad 0 \leqslant t \leqslant 24$$

where t is the number of hours after midnight.

Using the equation of the model and the answer to part (a),

(b) deduce the minimum temperature of the water during this day,

(1)

(c) find the time of day when the minimum temperature occurs for the 3rd time. Give your answer to the nearest minute.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

The following day the mean temperature of the water in the fish tank increased but the variation in temperature remained the same.

(d) State how the model could be refined to take this into account.

(1)