

11. Prove, using algebra, that for all positive integers n that are **not** divisible by 3

$$n^2 - 1$$

is divisible by 3

(4)

If n is not divisible by 3

or $n = 3k + 1$ for some integer k ,
 $n = 3k + 2$ (1 mark)

If $n = 3k + 1$,

$$\begin{aligned} n^2 - 1 &= (3k + 1)^2 - 1 \\ &= 9k^2 + 6k + 1 - 1 \\ &= 9k^2 + 6k \\ &= 3(3k^2 + 2k) \text{ which is divisible by 3 (1 mark)} \end{aligned}$$

If $n = 3k + 2$,

$$\begin{aligned} n^2 - 1 &= (3k + 2)^2 - 1 \\ &= 9k^2 + 12k + 4 - 1 \\ &= 9k^2 + 12k + 3 \\ &= 3(3k^2 + 4k + 1) \text{ which is divisible by 3 (1 mark)} \end{aligned}$$

So, we have proved by exhaustion that, if n is a positive integer which is not divisible by 3, $n^2 - 1$ is divisible by 3. (1 mark)