8. Lina has started a business selling candles.
In the first week of trading, Lina sold 150 candles.

In a model for future trading, Lina predicts that the number of candles sold each week will increase by a fixed number, d, so that

- $150 + d$ candles will be sold in week 2

- $150 + 2d$ candles will be sold in week 3

and so on.

(a) Use the model with $d = 10$ to find

    (i) the number of candles sold in week 30
    (ii) the **total** number of candles sold in the first 30 weeks of trading.

(4)

Lina must sell at least 15 000 candles in the first year of trading to make a profit.
Given that Lina's business does make a profit in the first year,

(b) find, according to the model, the smallest possible value of d.

(2)

(c) State a limitation of the model.

(c) the increase in candles produced each week, d, is unlikely to remain constant
(1 mark)

(1)

(a)(i) increasing by a fixed number each week, we have an arithmetic series with
$$a = 150$$
$$d = ?$$

Given $d = 10$, no. candles sold week $(n =) 30 = a + (n-1)d$
$$= 150 + (30-1)10$$
$$= 440 \quad (2 \text{ marks})$$

(a)(ii) Given $d = 10$, total no. candles sold in $(n = 30)$ weeks,

$$S_{30} = \frac{n}{2}(2a + (n-1)d) = \frac{30}{2}(2(150) + (30-1)10) \quad (1 \text{ mark})$$

$$= 15(300 + 290) = 8850 \quad (1 \text{ mark})$$

(b) A year has 52 weeks, so $n = 52$
For,
$$S_{52} = 15000 = \frac{52}{2}(2(150) + (52-1)d) \quad (1 \text{ mark})$$
$$15000 = 26(300 + 51d)$$
$$\Rightarrow d = \frac{15000 - 26(300)}{26(51)} = 5.429\ldots$$

d must be whole number. $d = 5$ will not be enough, so minimum $d = 6$ (1 mark)