

6. The curve C has equation

$$y = ax^3 + bx^2 + 12x + 2$$

where a and b are constants.

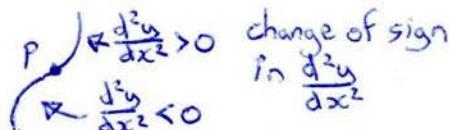
(a) Find, in terms of a and b ,

$$\begin{aligned} \text{(i)} \quad \frac{dy}{dx} & \stackrel{\text{(a)(i)}}{=} 3ax^{3-1} + 2bx^{2-1} + 12 \\ &= 3ax^2 + 2bx + 12 \quad (2 \text{ marks}) \\ \text{(ii)} \quad \frac{d^2y}{dx^2} & \stackrel{\text{(a)(ii)}}{=} 2(3)ax^{2-1} + 2b \\ &= 6ax + 2b \quad (1 \text{ mark}) \end{aligned}$$
(3)

Given that

- the point $P\left(\frac{3}{2}, \frac{13}{2}\right)$ lies on C (c) At P , $\frac{d^2y}{dx^2} = 0$, so P could be a point of inflection.
Just to left of P , $\frac{d^2y}{dx^2}(1.4) = 6(2)(1.4) + 2(-9) = -1.2 < 0$
Just to right of P , $\frac{d^2y}{dx^2}(1.6) = 6(2)(1.6) + 2(-9) = 1.2 > 0$ (1 mark)
- $\frac{d^2y}{dx^2} = 0$ at P

(b) find the value of a and the value of b .


 change of sign
 \Rightarrow point of inflection at P (1 mark) (2)

(c) Show that P is a point of inflection.

(b) At P ,

$$\begin{aligned} \frac{13}{2} &= a\left(\frac{3}{2}\right)^3 + b\left(\frac{3}{2}\right)^2 + 12\left(\frac{3}{2}\right) + 2 \\ \frac{13}{2} &= \frac{27a}{8} + \frac{9b}{4} + 18 + 2 \\ \frac{27a}{8} + \frac{9b}{4} &= \frac{13}{2} - 18 - 2 = -\frac{27}{2} \\ 27a + 18b &= -108 \\ 3a + 2b &= -12 \end{aligned}$$

At P ,

$$0 = 6a\left(\frac{3}{2}\right) + 2b \Rightarrow 9a + 2b = 0$$

Solving simultaneously,

$$\begin{aligned} 9a + 2b &= 0 \\ -(3a + 2b = -12) \end{aligned}$$

$$6a = 12$$

$$\Rightarrow \underline{\underline{a = 2}} \quad \& \quad 9(2) + 2b = 0 \Rightarrow \underline{\underline{b = -9}} \quad (2 \text{ marks})$$