

$$f(x) = px^3 - 7x^2 - x + q$$

where p and q are constants.

Given that $(2x + 1)$ is a factor of $f(x)$

(a) show that

$$8q - p = 10$$

(2)

Given that $(x - 1)$ is also a factor of $f(x)$

(b) find the value of p and the value of q .

(2)

(a) By Factor Theorem,

if $(2x + 1)$ is a factor of $f(x)$ then $f(-\frac{1}{2}) = 0$

$$\Rightarrow 0 = p(-\frac{1}{2})^3 - 7(-\frac{1}{2})^2 - (-\frac{1}{2}) + q \quad (1 \text{ mark})$$

$$0 = -\frac{1}{8}p - \frac{7}{4} + \frac{1}{2} + q$$

$$\times 8 \Rightarrow 0 = -p - 14 + 4 + 8q$$

$$\Rightarrow 8q - p = 10 \quad (1 \text{ mark})$$

(b) By Factor Theorem, $f(1)$ also $= 0$, so

$$0 = p(1)^3 - 7(1)^2 - (1) + q$$

$$0 = p - 7 - 1 + q$$

$$\Rightarrow q + p = 8$$

Solving simultaneously,

$$\begin{array}{r} 8q - p = 10 \\ + (q + p = 8) \\ \hline \end{array}$$

$$9q = 18 \Rightarrow q = 2 \quad (1 \text{ mark})$$

$$q + p = 8$$

$$2 + p = 8$$

$$\Rightarrow p = 6 \quad (1 \text{ mark})$$