

3.

(a) Trapezium Rule from Formula Book:

$$\int_a^b y \, dx \approx \frac{1}{2}h[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

where $h = \frac{b-a}{n}$

$$h = \frac{0.9 - (-0.6)}{5 \text{ (no. of trapezia)}} = 0.3 \text{ (1 mark)}$$

$$\int_{-0.6}^{0.9} f(x) \, dx$$

$$\approx \frac{1}{2}(0.3)[1.811 + 2.944 + 2(2.342 + 2.718 + 2.941 + 3.011)] \text{ (1 mark)}$$

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$.

The table shows corresponding values of x and y for this curve between $x = -0.6$ and $x = 0.9$

x	-0.6	-0.3	0	0.3	0.6	0.9
y	1.811	2.342	2.718	2.941	3.011	2.944

(a) contd

$$= 4.01685$$

$$= 4.02 \text{ 3sf (1 mark)}$$

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{-0.6}^{0.9} f(x) \, dx$$

Give your answer to 3 significant figures.

(3)

(b) State whether the answer to part (a) is an underestimate or overestimate of

(b) Sketch shows curve is concave between -0.6 and 0.9 .

justifying your answer.

$$\int_{-0.6}^{0.9} f(x) \, dx$$

This means top of each trapezium lies below the curve (see diagram) so area of trapezium less than area under curve.

So, trapezium rule gives underestimate here (1 mark) (1)

(c) Using the answer to part (a), find an estimate for

(c) Area under $2f(x)$ will be twice Area under $f(x)$
 $\approx 2 \times 4.02 = 8.04$ (1 mark)

$$\int_{-0.6}^{0.9} (8 - 2f(x)) \, dx$$

$$= \text{Area under rectangle} - \text{Area under curve } (2f(x))$$

$$\approx 12 - 8.04 = 3.96 \text{ (1 mark)}$$

(3)

Area $\int_{-0.6}^{0.9} 8 \, dx$ is rectangle $8 \times (0.9 - (-0.6)) = 8 \times 1.5 = 12$ (1 mark)