

2.

$$f(x) = x^3 - 5x^2 + \frac{3}{x} - 4 \quad x > 0$$

(a) Find $f'(x)$.Given that $f(0.5) = 0.875$ and $f(0.6) = -0.584$ (b) explain why the equation $f(x) = 0$ has a root in the interval $[0.5, 0.6]$

(1)

Given that this root is α and using 0.5 as a first approximation to α (c) apply the Newton-Raphson method once to $f(x)$ to find a second approximation to α .
Give your answer to 3 decimal places.

(2)

$$(a) f(x) = x^3 - 5x^2 + 3x^{-1} - 4$$

$$f'(x) = 3x^{3-1} - 2(5)x^{2-1} + (-1)3x^{-1-1} - 0$$

$$= 3x^2 - 10x - 3x^{-2}$$

$$= 3x^2 - 10x - \frac{3}{x^2} \quad (2 \text{ marks})$$

(b) there is a change of sign in the interval
and $f(x)$ is continuous (without asymptotes)
so there is a root in interval.

(1 mark)

(c) Given $x_0 = 0.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \quad \leftarrow \text{From Formula Book}$$

$$f(x_0) = f(0.5) = (0.5)^3 - 5(0.5)^2 + \frac{3}{0.5} - 4 = 0.875$$

$$f'(x_0) = f'(0.5) = 3(0.5)^2 - 10(0.5) - \frac{3}{(0.5)^2} = -16.25$$

$$x_1 = 0.5 - \frac{0.875}{-16.25} \quad (1 \text{ mark})$$

$$= 0.5538... = 0.554 \text{ 3dp} \quad (1 \text{ mark})$$