

14. (a) Express $\frac{3}{(2x-1)(x+1)}$ in partial fractions.

$$(a) \frac{3}{(2x-1)(x+1)} = \frac{A}{(2x-1)} + \frac{B}{(x+1)}$$

$$\Rightarrow 3 \equiv A(x+1) + B(2x-1) \quad (1 \text{ mark})$$

When $x = -1$, $3 = -3B \Rightarrow B = -1$ (1 mark)
 When $x = \frac{1}{2}$, $3 = \frac{3}{2}A \Rightarrow A = 2$ (1 mark)

(3)

When chemical A and chemical B are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

So,

$$\equiv \frac{2}{(2x-1)} - \frac{1}{(x+1)}$$
 (1 mark)

The total volume of oxygen produced, $V \text{ m}^3$, t hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)}$$

$$V \geq 0 \quad t \geq k$$

(b) Separating the Variables,

$$\int \frac{1}{V} dV = \int \frac{3}{(2t-1)(t+1)} dt$$
 (1 mark)

where k is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of 3 m^3 of oxygen had been produced,

(b) solve the differential equation to show that

(b) contd using result from (a),

$$\int \frac{1}{V} dV = \int \frac{2}{2t-1} dt - \int \frac{1}{t+1} dt$$
 (1 mark)

$$V = \frac{3(2t-1)}{(t+1)}$$

(b) contd $\ln V = \ln(2t-1) - \ln(t+1) + c$ (5)
 (1 mark) (c only needed on one side of equation.)

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

(c) (i) the **time delay** giving your answer in minutes,

(ii) the **limit** giving your answer in m^3

(2)

(b) contd. Given $V = 3$ when $t = 2$,
 $\ln(3) = \ln(2(2)-1) - \ln(2+1) + c$
 $\Rightarrow \ln(3) = \ln(3) - \ln(3) + c$
 $\Rightarrow c = \ln(3)$ (1 mark)

So,
 $\ln(V) = \ln(2t-1) - \ln(t+1) + \ln(3)$
 $\ln(V) = \ln\left(\frac{(2t-1) \times (3)}{t+1}\right)$

$$\Rightarrow V = \frac{3(2t-1)}{(t+1)} \quad (1 \text{ mark})$$

(c)(i) when $t = 0$, V is negative, which is meaningless.
 $V = 0$ when numerator
 $3(2t-1) = 0 \Rightarrow t = \frac{1}{2}$
 and then remains positive, so
 time delay is $\frac{1}{2}$ hour = 30 min (1 mark)

(c)(ii) as $t \rightarrow \infty$
 $V = \frac{3(2t-1)}{t+1} = \frac{6t-3}{t+1} \rightarrow 6 \text{ m}^3$
 (1 mark)