Question	Scheme	Marks	AOs
10(a)	Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3} \Rightarrow x=...$ Or substitutes $x=\frac{3}{2}$ into $\frac{5-3x}{2x-8}$	M1	3.1a
	$\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$	A1	1.1b
		(2)	
(b)	$\left(\frac{8x+5}{2x+3}=\right)4\pm\frac{...}{2x+3}$	M1	1.1b
	$\left(\frac{8x+5}{2x+3}=\right)4-\frac{7}{2x+3}$	A1	2.1
		(2)	
(c)	$0 \leqslant g^{-1}(x) \leqslant 4$	B1	2.2a
		(1)	
(d)	Attempts either boundary $f(0)=\frac{8\times0+5}{2\times0+3}$ **or** $f(4)=\frac{8\times4+5}{2\times4+3}$	M1	3.1a
	Attempts both boundaries $f(0)=\frac{8\times0+5}{2\times0+3}$ **and** $f(4)=\frac{8\times4+5}{2\times4+3}$	dM1	1.1b
	Range $\frac{5}{3} \leqslant fg^{-1}(x) \leqslant \frac{37}{11}$	A1	2.1
		(3)	
	Alternative by attempting $fg^{-1}(x)$		
	$g^{-1}(x)=\sqrt{16-x} \Rightarrow fg^{-1}(x)=\frac{8\sqrt{16-x}+5}{2\sqrt{16-x}+3}$ $fg^{-1}(0)=\frac{8\times4+5}{2\times4+3}$ **or** $fg^{-1}(16)=\frac{8\times0+5}{2\times0+3}$	M1	3.1a
	$fg^{-1}(0)=\frac{8\times4+5}{2\times4+3}$ **and** $fg^{-1}(16)=\frac{8\times0+5}{2\times0+3}$	dM1	1.1b
	Range $\frac{5}{3} \leqslant fg^{-1}(x) \leqslant \frac{37}{11}$	A1	2.1
		(3)	
			(8 marks)

Notes:

(a)

M1: Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3} \Rightarrow x=...$ You can condone poor algebra as long as they reach a value for x.

Alternatively attempt to substitute $x=\frac{3}{2}$ into $f^{-1}(x)=\frac{\pm5\pm3x}{\pm2x\pm8}$ or equivalent (may be in terms of y). Note that attempts to find e.g. $f'(x)$ or $\frac{1}{f(x)}$ which may be implied by values such as $\frac{6}{17}, \frac{17}{6}, \frac{7}{18}, \frac{18}{7}$ score M0

A1: Achieves $\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$. Do not be concerned what they call it, just look for the value e.g. $x=-\frac{1}{10}$ or just $-\frac{1}{10}$ is fine. Correct answer with no (or minimal) working scores both marks.

(b)

M1: Attempts to divide $8x+5$ by $2x+3$

Look for $4\pm\frac{...}{2x+3}$ where … is a constant or $8x+5=A(2x+3)+B$ with A or B correct (which may be in a fraction) or in a long division attempt and obtains a quotient of 4 or attempts to express the numerator in terms of the denominator e.g. $\frac{8x+5}{2x+3}=\frac{4(2x+3)+...}{2x+3}$

A1: A full and complete method showing $\frac{8x+5}{2x+3}=4-\frac{7}{2x+3}$ or $\frac{8x+5}{2x+3}=4+\frac{-7}{2x+3}$

Also allow for correct values e.g. $A = 4$, $B = -7$

Do not isw here e.g. if they obtain $A = 4$, $B = -7$ and then write $-7+\frac{4}{2x+3}$ score A0

(c)

B1: Deduces $0\leqslant g^{-1}(x)\leqslant 4$ o.e.

E.g. $0\leqslant y\leqslant 4$, $0\leqslant \text{range}\leqslant 4$, $g^{-1}(x)\leqslant 4$ and $g^{-1}(x)\geqslant 0$, $0\leqslant g^{-1}\leqslant 4$, $[0, 4]$

but not e.g. $0\leqslant x\leqslant 4$, $0\leqslant g(x)\leqslant 4$, $(0, 4)$

(d)

M1: Attempts either boundary. Look for either $f(0)=\frac{8\times0+5}{2\times0+3}$ **or** $f(4)=\frac{8\times4+5}{2\times4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times0+3}$ **or** $f(4)=4-\frac{7}{2\times4+3}$

dM1: Attempts both boundaries. Look for $f(0)=\frac{8\times0+5}{2\times0+3}$ **and** $f(4)=\frac{8\times4+5}{2\times4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times0+3}$ **and** $f(4)=4-\frac{7}{2\times4+3}$

A1: Correct answer written in the correct form.

E.g. $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant \text{range}\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant y\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}$ and $fg^{-1}(x)\geqslant\frac{5}{3}$

$\frac{5}{3}\leqslant fg^{-1}\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}\cap fg^{-1}(x)\geqslant\frac{5}{3}$, $\left[\frac{5}{3}, \frac{37}{11}\right]$ but not e.g. $\frac{5}{3}\leqslant x\leqslant\frac{37}{11}$

PTO for an alternative to (d)

(d) **Alternative:**

M1: Attempts $\mathrm{fg}^{-1}(x)$ and either boundary using $x = 0$ or $x = 16$

Look for either $\mathrm{fg}^{-1}(0)=\dfrac{8\times \mathrm{g}^{-1}(0)+5}{2\times \mathrm{g}^{-1}(0)+3}$ **or** $\mathrm{fg}^{-1}(16)=\dfrac{8\times \mathrm{g}^{-1}(16)+5}{2\times \mathrm{g}^{-1}(16)+3}$

Or uses (b) e.g. $\mathrm{fg}^{-1}(0)=4-\dfrac{7}{2\times \mathrm{g}^{-1}(0)+3}$ **or** $\mathrm{fg}^{-1}(16)=4-\dfrac{7}{2\times \mathrm{g}^{-1}(16)+3}$

The attempt at $\mathrm{fg}^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f

dM1: Attempts both boundaries. Look for $\mathrm{fg}^{-1}(0)=\dfrac{8\times \mathrm{g}^{-1}(0)+5}{2\times \mathrm{g}^{-1}(0)+3}$ **and** $\mathrm{fg}^{-1}(16)=\dfrac{8\times \mathrm{g}^{-1}(16)+5}{2\times \mathrm{g}^{-1}(16)+3}$

Or uses (b) e.g. $\mathrm{fg}^{-1}(0)=4-\dfrac{7}{2\times \mathrm{g}^{-1}(0)+3}$ **and** $\mathrm{fg}^{-1}(16)=4-\dfrac{7}{2\times \mathrm{g}^{-1}(16)+3}$

The attempt at $\mathrm{fg}^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f

A1: Correct answer written in the correct form with exact values.

E.g. $\dfrac{5}{3}\leqslant \mathrm{fg}^{-1}(x)\leqslant\dfrac{37}{11}$, $\dfrac{5}{3}\leqslant \text{range}\leqslant\dfrac{37}{11}$, $\dfrac{5}{3}\leqslant y\leqslant\dfrac{37}{11}$, $\mathrm{fg}^{-1}(x)\leqslant\dfrac{37}{11}$ and $\mathrm{fg}^{-1}(x)\geqslant\dfrac{5}{3}$

$\dfrac{5}{3}\leqslant \mathrm{fg}^{-1}\leqslant\dfrac{37}{11}$, $\mathrm{fg}^{-1}(x)\leqslant\dfrac{37}{11}\cap \mathrm{fg}^{-1}(x)\geqslant\dfrac{5}{3}$, $\left[\dfrac{5}{3},\ \dfrac{37}{11}\right]$ but not e.g. $\dfrac{5}{3}\leqslant x\leqslant\dfrac{37}{11}$

Note that the $\dfrac{37}{11}$ is sometimes obtained fortuitously from incorrect working so check working carefully.