Question	Scheme	Marks	AOs
14(a)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h} = 24$ or $\frac{\mathrm{d}h}{\mathrm{d}V} = \frac{1}{24}$	B1	3.1b
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}h}{\mathrm{d}V} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h}\frac{\mathrm{d}h}{\mathrm{d}t} \Rightarrow 0.48 - 0.1h = 24\frac{\mathrm{d}h}{\mathrm{d}t}$	M1	2.1
	$1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h^*$	A1*	1.1b
		(4)	
(b)	$1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h \Rightarrow \int \frac{1200}{24-5h}\mathrm{d}h = \int \mathrm{d}t$ $\Rightarrow e.g.\ \alpha \ln(24-5h) = t(+c)$ oe or $1200\frac{\mathrm{d}h}{\mathrm{d}t} = 24 - 5h \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}h} = \frac{1200}{24-5h}$ $\Rightarrow e.g.\ t(+c) = \alpha \ln(24-5h)$ oe	M1	3.1a
	$t = -240\ln(24-5h)(+c)$ oe	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240\ln(24-10) + c \Rightarrow c = ...(240\ln 14)$	M1	3.4
	$t = 240\ln(14) - 240\ln(24-5h)$	A1	1.1b
	$t = 240\ln\frac{14}{24-5h} \Rightarrow \frac{t}{240} = \ln\frac{14}{24-5h} \Rightarrow \mathrm{e}^{\frac{t}{240}} = \frac{14}{24-5h}$ $\Rightarrow 14\mathrm{e}^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = ...$	ddM1	2.1
	$h = 4.8 - 2.8\mathrm{e}^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}\mathrm{e}^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	**Examples:** • As $t \to \infty, \mathrm{e}^{-\frac{t}{240}} \to 0$ • When $h > 4.8$, $\frac{\mathrm{d}V}{\mathrm{d}t} < 0$ • Flow in = flow out at max h so $0.1h = 4.8 \rightarrow h = 4.8$ • As $\mathrm{e}^{-\frac{t}{240}} > 0,\ h < 4.8$ • $h = 5 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = -0.02$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{1}{1200}$ • $\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow h = 4.8$ • $h = 5 \Rightarrow 4.8 - 2.8\mathrm{e}^{-\frac{t}{240}} = 5 \Rightarrow \mathrm{e}^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If $h = 5$ the tank would be emptying so can never be full • The equation can't be solved when $h = 5$	A1	3.2a
		(2)	

(12 marks)

Notes

(a)

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}t}$ according to the model

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ according to the model

M1: Applies $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}h}{\mathrm{d}V}$ or equivalent correct formula with their $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}h}$ which may be implied by their working

A1*: Correct equation obtained with no errors

Note that: $\frac{\mathrm{d}V}{\mathrm{d}t}=0.48-0.1h\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{0.48-0.1h}{24}\Rightarrow 1200\frac{\mathrm{d}h}{\mathrm{d}t}=24-5h$ * scores

B1B0M0A0. There must be clear evidence where the "24" comes from and evidence of the correct chain rule being applied.

(b)

M1: Adopts a correct strategy by separating the variables correctly or rearranges to obtain $\frac{\mathrm{d}t}{\mathrm{d}h}$ correctly in terms of h **and** integrates to obtain $t=\alpha\ln\left(24-5h\right)\left(+c\right)$ or equivalent (condone missing brackets around the "24 – 5h") and + c not required for this mark.

A1: Correct equation in any form and + c not required. Do not condone missing brackets unless they are implied by subsequent work.

M1: Substitutes $t = 0$ and $h = 2$ to find their constant of integration (there must have been some attempt to integrate)

A1: Correct equation in any form

ddM1: Uses fully correct log work to obtain h in terms of t.

This depends on <u>both</u> previous method marks.

A1: Correct equation

Note that the marks may be earned in a different order e.g.:

$$t+c=-240\ln\left(24-5h\right)\Rightarrow-\frac{t}{240}+d=\ln\left(24-5h\right)\Rightarrow A\mathrm{e}^{-\frac{t}{240}}=24-5h$$

$$t=0,h=2\Rightarrow A=14\Rightarrow 14\mathrm{e}^{-\frac{t}{240}}=24-5h\Rightarrow h=4.8-2.8\mathrm{e}^{-\frac{t}{240}}$$

Score as M1 A1 as in main scheme then

M1: Correct work leading to $A\mathrm{e}^{\alpha t}=24-5h$ (must have a constant "A")

A1: $A\mathrm{e}^{-\frac{t}{240}}=24-5h$

ddM1: Uses $t = 0$, $h = 2$ in an expression of the form above to find A

A1: $h=4.8-2.8\mathrm{e}^{-\frac{t}{240}}$

(c)

M1: See scheme for some examples

A1: Makes a correct interpretation for their method.

There must be no incorrect working or contradictory statements.

This is not a follow through mark and if their equation in (b) is used it must be correct.