

11.

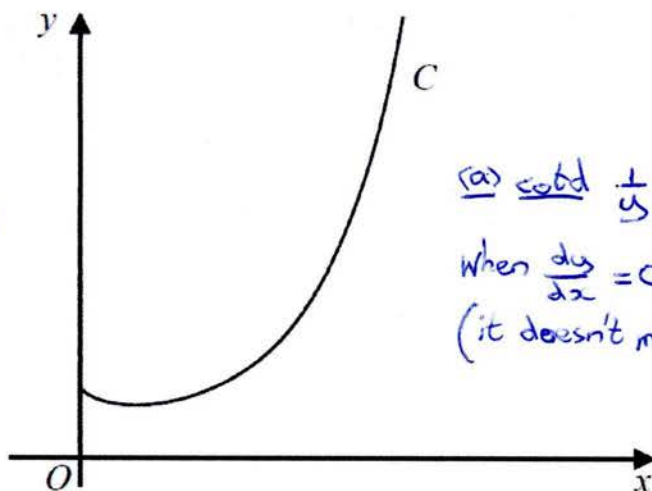
$$\text{(a)} \quad \ln y = \ln x^x$$

$$\ln y = x \ln x \quad (1 \text{ mark})$$

↓  
Implicit Differentiation  
↓

$$\frac{1}{y} \frac{dy}{dx} = 1 \ln x + \frac{x}{x}$$

$$\frac{1}{y} \frac{dy}{dx} = \ln x + 1 \quad (2 \text{ marks})$$



$$\text{(a) contd} \quad \frac{1}{y} \frac{dy}{dx} = \ln x + 1$$

When  $\frac{dy}{dx} = 0$ ,  $\ln x + 1 = 0$  (1 mark)  
(it doesn't matter about  $\frac{1}{y}$ )

$$\ln x = -1$$

$$\Rightarrow x = e^{-1} \quad (1 \text{ mark})$$

Figure 8

Figure 8 shows a sketch of the curve  $C$  with equation

$$y = x^x, \quad x > 0.$$

(a) Find, by firstly taking logarithms, the  $x$  coordinate of the turning point of  $C$ .

(Solutions based entirely on graphical or numerical methods are not acceptable.)

$$\text{(b)} \quad 2 = x^x \Rightarrow 2 - x^x = 0 \quad (5)$$

The point  $P(\alpha, 2)$  lies on  $C$ .

$$2 - 1.5^{1.5} = 0.16288...$$

(b) Show that  $1.5 < \alpha < 1.6$ .

$$2 - 1.6^{1.6} = -0.12125...$$

change of sign and function is continuous (2 marks)  
so  $1.5 < \alpha < 1.6$

A possible iteration formula that could be used in an attempt to find  $\alpha$  is

$$x_{n+1} = 2x_n^{1-x_n}.$$

Using this formula with  $x_1 = 1.5$ ,

$$\text{(c)} \quad x_1 = 1.5$$

$$x_2 = 1.63299...$$

$$x_3 = 1.46626...$$

$$x_4 = 1.67313...$$

$$\left. \begin{array}{l} x_2 = 1.63299... \\ x_3 = 1.46626... \\ x_4 = 1.67313... \end{array} \right\} 2[\text{Ans}]^{1-[\text{Ans}]} \Rightarrow x_4 = 1.673 \text{ 3dp} \quad (2)$$

(d) describe the long-term behaviour of  $x_n$ .

(d)  $x_n$  does not converge, but oscillates, diverging until it settles oscillating between 1 and 2. (Found by repeating iteration on calculator). (2 marks)

(Total for Question 11 is 11 marks)