14. (*a*) Use the substitution $u = 6 - \sqrt{h}$ to show that

$$\int \frac{1}{6 - \sqrt{h}} \, dh = -12 \ln |6 - \sqrt{h}| - 2\sqrt{h} + k,$$

where k is a constant

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{dh}{dt} = \frac{t^{0.25}(6 - \sqrt{h})}{20},$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(*b*) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted. According to the model,

(*c*) calculate the time this tree would take to reach a height of 15 metres, giving your answer to 3 significant figures.

(7)

(Total for Question 14 is 15 marks)