Question	Scheme	Marks	AOs
13 (a)	States or uses $6=\pi r^2h+\frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow h=\frac{6}{\pi r^2}-\frac{2}{3}r,\ \pi h=\frac{6}{r^2}-\frac{2}{3}\pi r,\ \pi rh=\frac{6}{r}-\frac{2}{3}\pi r^2,\ rh=\frac{6}{\pi r}-\frac{2}{3}r^2$		
	$A=\pi r^2+2\pi rh+2\pi r^2\ \{\Rightarrow A=3\pi r^2+2\pi rh\}$		
	$A=2\pi r^2+2\pi r\left(\frac{6}{\pi r^2}-\frac{2}{3}r\right)+\pi r^2$	M1	3.1a
		A1	1.1b
	$A=3\pi r^2+\frac{12}{r}-\frac{4}{3}\pi r^2\ \Rightarrow\ A=\frac{12}{r}+\frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A=12r^{-1}+\frac{5}{3}\pi r^2\Rightarrow\right\}\ \frac{\mathrm{d}A}{\mathrm{d}r}=-12r^{-2}+\frac{10}{3}\pi r$	M1	3.4
		A1	1.1b
	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r}=0\Rightarrow\right\}\ -\frac{12}{r^2}+\frac{10}{3}\pi r=0\ \Rightarrow\ -36+10\pi r^3=0\Rightarrow r^{\pm3}=...\left\{=\frac{18}{5\pi}\right\}$	M1	2.1
	$r=1.046447736...\Rightarrow r=$ 1.05 (m) (3 sf) or awrt 1.05 (m)	A1	1.1b
	Note: Give final A1 for correct exact values for r	**(4)**	
(c)	$A_{\min}=\frac{12}{(1.046...)}+\frac{5}{3}\pi(1.046...)^2$	M1	3.4
	$\{A_{\min}=17.20...\ \Rightarrow\}\ A=17\ (\mathrm{m}^2)$ or $A=$ awrt 17 (m^2)	A1ft	1.1b
		(2)	
			(10 marks)

Notes for Question 13

(a)	
B1:	See scheme
M1:	Complete process of substituting their $h=...$ or $\pi h=...$ or $\pi rh=...$ or $rh=...$, where $'...'=\mathrm{f}(r)$ into an expression for the surface area which is of the form $A=\lambda\pi r^2+\mu\pi rh;\ \lambda,\mu\neq0$
A1:	Obtains correct simplified or un-simplified $\{A=\}\ 2\pi r^2+2\pi r\left(\frac{6}{\pi r^2}-\frac{2}{3}r\right)+\pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
Note:	Condone the lack of $A=...$ or $S=...$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r}+\mu r^2$ to give $\alpha r^{-2}+\beta r$; $\lambda,\mu,\alpha,\beta\neq0$
A1:	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r}=\right\}\ -12r^{-2}+\frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ and rearranges to give $r^{\pm3}=k,\ k\neq0$ (**Note:** k can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36+10\pi r^3=0\rightarrow r=\left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r=$ awrt 1.05 (ignoring units) or $r=$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r=$ 1.05 (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for r. E.g. $r=\left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

<table>
<tr><th colspan="2">Notes for Question 13 Continued</th></tr>
<tr><td>Note:</td><td>Give final M0 A0 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464$</td></tr>
<tr><td>Note:</td><td>Give final M1 A1 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464... \Rightarrow r = 1.0464...$</td></tr>
<tr><td>(c)</td><td></td></tr>
<tr><td>M1:</td><td>Substitutes their $r = 1.046...$, found from solving $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ in part (b), into the model with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$</td></tr>
<tr><td>Note:</td><td>Give M0 for substituting their r which has been found from solving $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=0$ or from using $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$ into the model with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$</td></tr>
<tr><td>A1ft:</td><td>$\{A=\}$ 17 or $\{A=\}$ awrt 17 (ignoring units)</td></tr>
<tr><td>Note:</td><td>You can only follow through on values of r for $0.6 \le$ their $r \le 1.3$ (and where their r has been found from solving $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ in part (b))
<table>
<tr><th>r</th><th>A</th><th>A
(nearest integer)</th></tr>
<tr><td>0.6</td><td>21.88495…</td><td>awrt 22</td></tr>
<tr><td>0.7</td><td>19.70849…</td><td>awrt 20</td></tr>
<tr><td>0.8</td><td>18.35103…</td><td>awrt 18</td></tr>
<tr><td>0.9</td><td>17.57448…</td><td>awrt 18</td></tr>
<tr><td>1.0</td><td>17.23598…</td><td>awrt 17</td></tr>
<tr><td>1.1</td><td>17.24463…</td><td>awrt 17</td></tr>
<tr><td>1.2</td><td>17.53982…</td><td>awrt 18</td></tr>
<tr><td>1.3</td><td>18.07958…</td><td>awrt 18</td></tr>
<tr><td>1.05</td><td>17.20124…</td><td>awrt 17</td></tr>
<tr><td>1.04644…</td><td>17.20105…</td><td>awrt 17</td></tr>
</table></td></tr>
<tr><td>Note:</td><td>Give M1 A1 for $A=17$ (m^2) or $A=$ awrt 17 (m^2) from no working</td></tr>
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