Question	Scheme	Marks	AOs
12	$\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$		
(a) Way 1	$\{\text{LHS} = \}\ \dfrac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \dfrac{\cos(3\theta - \theta)}{\sin\theta\cos\theta}\ \left\{= \dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	$\{\text{LHS} = \}\ \dfrac{\cos 2\theta\cos\theta - \sin 2\theta\sin\theta}{\sin\theta} + \dfrac{\sin 2\theta\cos\theta + \cos 2\theta\sin\theta}{\cos\theta}$		
	$= \dfrac{\cos 2\theta\cos^2\theta - \sin 2\theta\sin\theta\cos\theta + \sin 2\theta\cos\theta\sin\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \dfrac{\cos 2\theta(\cos^2\theta + \sin^2\theta)}{\sin\theta\cos\theta}\ \left\{= \dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 3	$\{\text{RHS} = \}\ \dfrac{2\cos 2\theta}{\sin 2\theta} = \dfrac{2\cos(3\theta - \theta)}{\sin 2\theta} = \dfrac{2(\cos 3\theta\cos\theta + \sin 3\theta\sin\theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$= \dfrac{2(\cos 3\theta\cos\theta + \sin 3\theta\sin\theta)}{2\sin\theta\cos\theta}$	dM1	1.1b
	$= \dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta}$ *	A1 *	2.1
		(4)	
(b) Way 1	$\left\{\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow 2\left(\dfrac{1}{\tan 2\theta}\right) = 4$	M1	1.1b
	Rearranges to give $\tan 2\theta = k;\ k \neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{90° < \theta < 180°,\ \tan 2\theta = \dfrac{1}{2} \Rightarrow\right\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
(b) Way 2	$\left\{\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow \dfrac{2}{\tan 2\theta} = 4$	M1	1.1b
	$\dfrac{2}{\left(\dfrac{2\tan\theta}{1 - \tan^2\theta}\right)} = 4 \Rightarrow 2(1 - \tan^2\theta) = 8\tan\theta$ $\Rightarrow \tan^2\theta + 4\tan\theta - 1 = 0 \Rightarrow \tan\theta = \dfrac{-4 \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$ $\{\Rightarrow \tan\theta = -2 \pm \sqrt{5}\} \Rightarrow \tan\theta = k;\ k \neq 0 \Rightarrow$ applies $\arctan k$	dM1	1.1b
	$\{90° < \theta < 180°,\ \tan\theta = -2 - \sqrt{5} \Rightarrow\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
		(7 marks)	

Notes for Question 12	
(a)	**Way 1 and Way 2**
M1:	Correct valid method forming a common denominator of $\sin\theta\cos\theta$ i.e. correct process of $\dfrac{(...)\cos\theta+(...)\sin\theta}{\cos\theta\sin\theta}$
A1:	Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta-\theta)$ or $\cos 2\theta$
dM1:	**dependent on the previous M mark** Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$
A1*	Correct proof
Note:	Writing $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta}{\sin\theta\cos\theta} + \dfrac{\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ is considered a correct valid method of forming a common denominator of $\sin\theta\cos\theta$ for the 1st M1 mark
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$
Note:	Allow 2nd M1 for stating a correct $\sin 2\theta = 2\sin\theta\cos\theta$ and for attempting to apply it to the common denominator $\sin\theta\cos\theta$
(a)	**Way 3**
M1:	Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta\cos\theta \pm \sin 3\theta\sin\theta$
A1:	Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta\cos\theta + \sin 3\theta\sin\theta$
dM1:	**dependent on the previous M mark** Applies $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to their denominator
A1*:	Correct proof
Note:	Allow 1st M1 1st A1 (together) for any of LHS $\rightarrow \dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ or LHS $\rightarrow \dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ or LHS $\rightarrow \cos 2\theta(\cot\theta+\tan\theta)$ or LHS $\rightarrow \cos 2\theta\left(\dfrac{1+\tan^2\theta}{\tan\theta}\right)$ (i.e. where $\cos 2\theta$ has been factorised out)
Note:	Allow 1st M1 1st A1 for progressing as far as LHS $= ... = \cot x - \tan x$
Note:	The following is a correct alternative solution $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\frac{1}{2}(\cos 4\theta+\cos 2\theta)-\frac{1}{2}(\cos 4\theta-\cos 2\theta)}{\sin\theta\cos\theta}$ $= \dfrac{\cos 2\theta}{\sin\theta\cos\theta} = \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *
Note:	E.g. going from $\dfrac{\cos 2\theta\cos^2\theta - \sin 2\theta\sin\theta\cos\theta + \sin 2\theta\cos\theta\sin\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ to $\dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ with no intermediate working is 1st A0

Notes for Question 12 Continued	
(b)	**Way 1**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Rearranges to give $\tan 2\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$
Note:	Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
Note:	1^{st} M1 can be implied by seeing $\tan 2\theta = \frac{1}{2}$
Note:	Condone 2^{nd} M1 for applying $\frac{1}{2}\arctan\left(\frac{1}{2}\right)$ $\{= 13.28...\}$
(b)	**Way 2**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Applies $\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta}$, forms and uses a correct method for solving a 3TQ to give $\tan\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$
Note:	Give M1 dM1 A1 for no working leading to $\theta =$ awrt $103.3°$ and no other solutions
Note:	Give M1 dM1 A0 for no working leading to $\theta =$ awrt $103.3°$ and other solutions which can be either outside or inside the range $90° < \theta < 180°$