| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **13** | $x = 6\cos t\,,\; y = 5\sin 2t;\quad 0 \leqslant t \leqslant \frac{\pi}{2}$ | | |
| | $\left\{\int y\frac{\mathrm{d}x}{\mathrm{d}t}\{\mathrm{d}t\}\right\} = \int (5\sin 2t)(-6\sin t)\{\mathrm{d}t\}$ | M1 | 2.1 |
| | | A1 | 1.1b |
| | $= \int (5(2\sin t\cos t))(-6\sin t)\{\mathrm{d}t\}$ | M1 | 1.1b |
| | $= -60\int \sin^2 t\cos t\{\mathrm{d}t\}$ | | |
| | $= -60\left[\frac{1}{3}\sin^3 t\right] \quad \left\{= -20\left[\sin^3 t\right]\right\}$ | M1 | 3.1a |
| | | A1 | 1.1b |
| | $\left\{\text{Limits: } x = 0 \Rightarrow 0 = 6\cos t \Rightarrow t = \frac{\pi}{2};\;\; x = 3 \Rightarrow 3 = 6\cos t \Rightarrow t = \frac{\pi}{3}\right\}$ | | |
| | $\text{Area }(R) = \int_0^3 y\,\mathrm{d}x = -20\left[\sin^3 t\right]_{\frac{\pi}{2}}^{\frac{\pi}{3}} = -20\left(\sin^3\left(\frac{\pi}{3}\right) - \sin^3\left(\frac{\pi}{2}\right)\right)$ | M1 | 1.1b |
| | $= -20\left(\left(\frac{\sqrt{3}}{2}\right)^3 - 1\right) = -20\left(\frac{3}{8}\sqrt{3} - 1\right) = 20 - \frac{15}{2}\sqrt{3}$ * | A1* | 2.1 |
| | | **(7)** | |
| | | **(7 marks)** | |

**Question 13 Notes:**

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|---|---|
| **M1:** | Begins proof by applying a full method of $\int y\frac{\mathrm{d}x}{\mathrm{d}t}\{\mathrm{d}t\}$ to give $\int (5\sin 2t)\left(\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}\right)\{\mathrm{d}t\}$. |
| **A1:** | $\int (5\sin 2t)(-6\sin t)\{\mathrm{d}t\}$. |
| **M1:** | Applies $\sin 2t \equiv 2\sin t\cos t$ to achieve an integral of the form $\pm K\int \sin^2 t\cos t\{\mathrm{d}t\};\;\; K \neq 0$, which may be un-simplified or simplified |
| **M1:** | Applies parametric integration to achieve an integral of the form $\pm K\int \sin^2 t\cos t\{\mathrm{d}t\};\;\; K \neq 0$, followed by a correct integration strategy of "reverse chain rule" or "integration by substitution" to give $\int \sin^2 t\cos t\{\mathrm{d}t\}$ in the form $\pm\lambda\sin^3 t;\, \lambda \neq 0$ or $\pm\lambda u^3;\, \lambda \neq 0$ where $u = \sin t$ |
| **A1:** | $\sin^2 t\cos t \to \frac{1}{3}\sin^3 t$ or $\sin^2 t\cos t \to \frac{1}{3}u^3$ where $u = \sin t$ |
| **M1:** | Applies limits of $t = \frac{\pi}{3}$ and $t = \frac{\pi}{2}$ to an integrated expression of the form $\pm\alpha\sin^3 t;\, \alpha \neq 0$ and subtracts either way round |
| **A1*:** | Correctly uses their limits to show that the area of $R$ is $20 - \frac{15}{2}\sqrt{3}$ |