$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Question	Scheme	Marks	AOs
(i) $f(x) = (x-2)(x^{2}-4x-1)$ (2) (i) $f(x) = (x-2)(x^{2}-4x-1)$ (i) $f(x) = (x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (i) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (j) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 1 = 0 \text{ or } x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ (k) $f(x) = \frac{(x-2)^{2} - 4 - 4 \pm \sqrt{16 -$	4	$f(x) = x^3 - 6x^2 + 7x + 2, x \in \mathbb{R}$		
(i) (ii) (ii) $\frac{\{\text{Note: } (x-2)=0 \Rightarrow x_{\varrho} = 2 \text{ is known and at } P, R, (x^{2}-4x-1)=0\}}{\{(x-2)^{2}-4-1=0 \text{ or } x=\frac{4\pm\sqrt{16-4(1)(-1)}}{2(1)}}{2(1)}}$ M1 1.1b $\Rightarrow x_{p} = 2 - \sqrt{5} \text{ and } x_{R} = 2 + \sqrt{5}$ A1 1.1b (2) (c) $\frac{\sin^{3}\theta - 6\sin^{2}\theta + 7\sin\theta + 2 = 0, -\pi \le \theta \le 12\pi,}{\text{Deduces that there are 14 real solutions for } -\pi \le \theta \le 12\pi}$ B1 2.2a Correct justification. E.g. Both • $\sin \theta = 2 \text{ and } \sin \theta = 2 + \sqrt{5} = 4.236 \text{ have no real solutions and either}}$ B1 2.2a Correct justification. E.g. Both • $\sin \theta = 2 - \sqrt{5} = -0.236 \text{ has 2 real solutions for each interval of } 2\pi. \text{ So there are 12 real solutions in the interval } [-\pi, 11\pi] \text{ and } 2 \text{ real solutions in the interval } [-\pi, 11\pi] \text{ and } 2 \text{ real solutions in the interval } [-2\pi, 12\pi] \text{ and no real solutions in the interval } [-2\pi, -\pi]$ • $\sin \theta = 2 - \sqrt{5} = -0.236 \text{ has two real solutions for each interval of } 2\pi. \text{ So there are 12 real solutions in the interval } [-2\pi, 12\pi] \text{ and } 2 \text{ real solutions in the interval } [-2\pi, 12\pi] \text{ and } 2 \text{ real solutions in the interval } [-2\pi, -\pi]$ • $\sin \theta = 2 - \sqrt{5} = -0.236 \text{ has two real solutions for each interval of } 2\pi. \text{ So there are 12 real solutions in the interval } [-2\pi, 12\pi] \text{ and } 10 \text{ correct } [-\pi, 0], [\pi, 2\pi], [3\pi, 4\pi], [5\pi, 6\pi], [7\pi, 8\pi], [9\pi, 10\pi] \text{ and } [11\pi, 12\pi]$	(a)	$f(x) = (x - 2)(x^2 - 4x - 1)$	M1	2.2a
(b) (i), (ii) {(Note: $(x-2) = 0 \Rightarrow x_{\varrho} = 2$ is known and at $P, R, (x^2 - 4x - 1) = 0$ {($x-2)^2 - 4 - 1 = 0$ or $x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$ M1 1.1b $\Rightarrow x_P = 2 - \sqrt{5}$ and $x_R = 2 + \sqrt{5}$ A1 1.1b (2) (c) $\sin^3 \theta - 6\sin^2 \theta + 7\sin \theta + 2 = 0, -\pi \le \theta \le 12\pi,$ B1 2.2a Correct justification. E.g. Both • $\sin \theta = 2$ and $\sin \theta = 2 + \sqrt{5} = 4.236$ have no real solutions and either • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[0, 12\pi]$ and 2 real solutions in the interval $[-\pi, 0]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[0, 12\pi]$ and 2 real solutions in the interval $[-\pi, 11\pi]$ and 2 real solutions in the interval $[-\pi, 12\pi]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[-\pi, 11\pi]$ and 2 real solutions in the interval $[-\pi, 0]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 14 real solutions in the interval $[-\pi, 11\pi]$ and $2 real solutions in the interval [-2\pi, -\pi]• \sin \theta = 2 - \sqrt{5} = -0.236 has two real solutions in each of[-\pi, 0], [\pi, 2\pi], [3\pi, 4\pi], [5\pi, 6\pi], [7\pi, 8\pi], [9\pi, 10\pi] and[11\pi, 12\pi]$		$\Gamma(\lambda) = (\lambda - 2)(\lambda - 4\lambda - 1)$	A1	1.1b
(i), (ii) $\begin{array}{c c c c c c c c c c c c c c c c c c c $			(2)	
$\Rightarrow x_{p} = 2 - \sqrt{5} \text{ and } x_{k} = 2 + \sqrt{5}$ A1 1.1b (2) (c) $\frac{\sin^{3}\theta - 6\sin^{2}\theta + 7\sin\theta + 2 = 0, -\pi \le \theta \le 12\pi, B1 2.2a}{\text{Deduces that there are 14 real solutions for } -\pi \le \theta \le 12\pi, B1 2.2a}$ Correct justification. E.g. Both • $\sin \theta = 2$ and $\sin \theta = 2 + \sqrt{5} = 4.236$ have no real solutions and either • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[0, 12\pi]$ and 2 real solutions in the interval $[-\pi, 0]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions for each interval of 2π . So there are 12 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[-\pi, 11\pi]$ and 2 real solutions in the interval $[11\pi, 12\pi]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 14 real solutions in the interval $[-2\pi, 12\pi]$ and no real solutions in the interval $[-2\pi, -\pi]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has two real solutions in each of $[-\pi, 0], [\pi, 2\pi], [3\pi, 4\pi], [5\pi, 6\pi], [7\pi, 8\pi], [9\pi, 10\pi]$ and $[11\pi, 12\pi]$	(b)			
(c) $\sin^3 \theta - 6\sin^2 \theta + 7\sin \theta + 2 = 0, -\pi \leq \theta \leq 12\pi,$ (2)Deduces that there are 14 real solutions for $-\pi \leq \theta \leq 12\pi$ B12.2aCorrect justification. E.g. BothBothB12.2a \circ sin $\theta = 2$ and sin $\theta = 2 + \sqrt{5} = 4.236$ have no real solutions and either \circ sin $\theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[0, 12\pi]$ and 2 real solutions in the interval $[-\pi, 0]$ B1ft2.4 \circ sin $\theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[-\pi, 11\pi]$ and 2 real solutions in the interval $[11\pi, 12\pi]$ B1ft2.4 \circ sin $\theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 14 real solutions in the interval $[-\pi, 11\pi]$ and 2 real solutions in the interval $[-\pi, 12\pi]$ and no real solutions in the interval $[-2\pi, -\pi]$ B1ft2.4	(i), (ii)	$(x-2)^2 - 4 - 1 = 0$ or $x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$	M1	1.1b
(c) $\frac{\sin^{3}\theta - 6\sin^{2}\theta + 7\sin\theta + 2 = 0, -\pi \leq \theta \leq 12\pi,}{\text{Deduces that there are 14 real solutions for } -\pi \leq \theta \leq 12\pi}$ B1 2.2a Correct justification. E.g. Both • $\sin \theta = 2$ and $\sin \theta = 2 + \sqrt{5} = 4.236$ have no real solutions and either • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[0, 12\pi]$ and 2 real solutions in the interval $[-\pi, 0]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[-\pi, 11\pi]$ and 2 real solutions in the interval $[11\pi, 12\pi]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 14 real solutions in the interval $[-2\pi, 11\pi]$ and $2 \text{ real solutions in the interval } [12\pi, 12\pi]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has two real solutions in each of $[-\pi, 0], [\pi, 2\pi], [3\pi, 4\pi], [5\pi, 6\pi], [7\pi, 8\pi], [9\pi, 10\pi] and$ $[11\pi, 12\pi]$		$\Rightarrow x_P = 2 - \sqrt{5}$ and $x_R = 2 + \sqrt{5}$	A1	1.1b
Deduces that there are 14 real solutions for $-\pi \le \theta \le 12\pi$ B12.2aDeduces that there are 14 real solutions for $-\pi \le \theta \le 12\pi$ B12.2aCorrect justification. E.g. Both• $\sin \theta = 2$ and $\sin \theta = 2 + \sqrt{5} = 4.236$ have no real solutions and either• $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[0, 12\pi]$ and 2 real solutions in the interval $[-\pi, 0]$ B1ft2.4• $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[-\pi, 11\pi]$ and 2 real solutions in the interval $[11\pi, 12\pi]$ B1ft2.4• $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 14 real solutions in the interval $[-2\pi, 11\pi]$ and 2π . So there are 14 real solutions in the interval $[-2\pi, 12\pi]$ and no real solutions in the interval $[-2\pi, -\pi]$ B1ft2.4• $\sin \theta = 2 - \sqrt{5} = -0.236$ has two real solutions in each of $[-\pi, 0], [\pi, 2\pi], [3\pi, 4\pi], [5\pi, 6\pi], [7\pi, 8\pi], [9\pi, 10\pi]$ and $[11\pi, 12\pi]$ 111111111111111111111111111111111			(2)	
Correct justification. E.g. Both • $\sin \theta = 2$ and $\sin \theta = 2 + \sqrt{5} = 4.236$ have no real solutions and either • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[0, 12\pi]$ and 2 real solutions in the interval $[-\pi, 0]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 12 real solutions in the interval $[-\pi, 11\pi]$ and 2 real solutions in the interval $[11\pi, 12\pi]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has 2 real solutions for each interval of 2π . So there are 14 real solutions in the interval $[-2\pi, 12\pi]$ and no real solutions in the interval $[-2\pi, -\pi]$ • $\sin \theta = 2 - \sqrt{5} = -0.236$ has two real solutions in each of $[-\pi, 0], [\pi, 2\pi], [3\pi, 4\pi], [5\pi, 6\pi], [7\pi, 8\pi], [9\pi, 10\pi]$ and $[11\pi, 12\pi]$	(c)	$\sin^3 \theta - 6\sin^2 \theta + 7\sin \theta + 2 = 0, -\pi \leqslant \theta \leqslant 12\pi,$		
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(2)		 Both sin θ = 2 and sin θ = 2 + √5 = 4.236 have no real solutions and either sin θ = 2 - √5 = -0.236 has 2 real solutions for each interval of 2π. So there are 12 real solutions in the interval [0, 12π] and 2 real solutions in the interval [-π, 0] sin θ = 2 - √5 = -0.236 has 2 real solutions for each interval of 2π. So there are 12 real solutions in the interval [-π, 11π] and 2 real solutions in the interval [11π, 12π] sin θ = 2 - √5 = -0.236 has 2 real solutions for each interval of 2π. So there are 14 real solutions in the interval [-2π, 12π] and no real solutions in the interval [-2π, -π] sin θ = 2 - √5 = -0.236 has two real solutions in each of [-π, 0], [π, 2π], [3π, 4π], [5π, 6π], [7π, 8π], [9π, 10π] and 	B1ft	2.4
			(2)	

(6 marks)

Question 4 Notes:

(a) M1: Deduces (x-2) is a factor of f(x) and attempts to find a quadratic factor of f(x) by either equating coefficients or by algebraic long division $(x-2)(x^2-4x-1)$ A1: **(b)** (i), (ii) M1: Correct method (i.e. completing the square or applying the quadratic formula) to solve a 3TQ. Note: M1 can be given here for at least one of either $2 - \sqrt{5}$ or $2 + \sqrt{5}$ written down in part (b). A1: Finds and identifies the correct exact x coordinate of P and the correct exact x coordinate of R (c) **B1**: Correct deduction of 14 (real solutions) **B1**: See scheme