

Question	Scheme	Marks	AOs
4	$f(x) = x^3 - 6x^2 + 7x + 2, \quad x \in \mathbb{R}$		
(a)	$f(x) = (x - 2)(x^2 - 4x - 1)$	M1	2.2a
		A1	1.1b
		(2)	
(b) (i), (ii)	{Note: $(x - 2) = 0 \Rightarrow x_Q = 2$ is known and at $P, R, (x^2 - 4x - 1) = 0$ }		
	$(x - 2)^2 - 4 - 1 = 0 \quad \text{or} \quad x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$	M1	1.1b
	$\Rightarrow x_P = 2 - \sqrt{5} \text{ and } x_R = 2 + \sqrt{5}$	A1	1.1b
		(2)	
(c)	$\sin^3 \theta - 6\sin^2 \theta + 7\sin \theta + 2 = 0, \quad -\pi \leq \theta \leq 12\pi,$		
	Deduces that there are 14 real solutions for $-\pi \leq \theta \leq 12\pi$	B1	2.2a
	Correct justification. E.g. Both <ul style="list-style-type: none"><li><math>\sin \theta = 2</math> and <math>\sin \theta = 2 + \sqrt{5} = 4.236\dots</math> have no real solutions and either</li><li><math>\sin \theta = 2 - \sqrt{5} = -0.236\dots</math> has 2 real solutions for each interval of <math>2\pi</math>. So there are 12 real solutions in the interval <math>[0, 12\pi]</math> and 2 real solutions in the interval <math>[-\pi, 0]</math></li><li><math>\sin \theta = 2 - \sqrt{5} = -0.236\dots</math> has 2 real solutions for each interval of <math>2\pi</math>. So there are 12 real solutions in the interval <math>[-\pi, 11\pi]</math> and 2 real solutions in the interval <math>[11\pi, 12\pi]</math></li><li><math>\sin \theta = 2 - \sqrt{5} = -0.236\dots</math> has 2 real solutions for each interval of <math>2\pi</math>. So there are 14 real solutions in the interval <math>[-2\pi, 12\pi]</math> and no real solutions in the interval <math>[-2\pi, -\pi]</math></li><li><math>\sin \theta = 2 - \sqrt{5} = -0.236\dots</math> has two real solutions in each of <math>[-\pi, 0], [\pi, 2\pi], [3\pi, 4\pi], [5\pi, 6\pi], [7\pi, 8\pi], [9\pi, 10\pi]</math> and <math>[11\pi, 12\pi]</math></li></ul>	B1ft	2.4
		(2)	

(6 marks)

#### Question 4 Notes:

(a)

**M1:** Deduces  $(x - 2)$  is a factor of  $f(x)$  and attempts to find a quadratic factor of  $f(x)$  by either equating coefficients or by algebraic long division

**A1:**  $(x - 2)(x^2 - 4x - 1)$

(b)

(i), (ii)

**M1:** Correct method (i.e. completing the square or applying the quadratic formula) to solve a 3TQ.

**Note:** M1 can be given here for at least one of either  $2 - \sqrt{5}$  or  $2 + \sqrt{5}$  written down in part (b).

**A1:** Finds and identifies the correct exact  $x$  coordinate of  $P$  and the correct exact  $x$  coordinate of  $R$

(c)

**B1:** Correct deduction of 14 (real solutions)

**B1:** See scheme