Question	Scheme	Marks	AOs
12	$1-\cos 2\theta \equiv \tan\theta\sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n\in\mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x-5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x-5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x-5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x=0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x-3\tan x-5)\tan x=0$ **or** $(1+\tan^2 x-3\tan x-5)(1-\cos 2x)=0$ **or** $1+\tan^2 x-5 = 3\tan x$	M1	2.1
	$\tan^2 x-3\tan x-4=0$	A1	1.1b
	$(\tan x-4)(\tan x+1)=0 \Rightarrow \tan x = \ldots$	M1	1.1b
	$x = -\frac{\pi}{4},\ 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Notes for Question 12

(a)	**Way 1**
M1:	Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta-\sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta-1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and
	$\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS = $2\sin^2\theta$ and RHS = $2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Notes for Question 12 Continued	
(b)	
B1:	Deduces that the given equation yields a solution $x = 0$
M1:	For using the key step of $\sec^2 x = 1 + \tan^2 x$ and cancels/factorises out $\tan x$ or $(1 - \cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$
Note:	Allow the use of $\pm\sec^2 x = \pm 1 \pm \tan^2 x$ for M1
A1:	Correct 3TQ in $\tan x$. E.g. $\tan^2 x - 3\tan x - 4 = 0$
Note:	E.g. $\tan^2 x - 4 = 3\tan x$ or $\tan^2 x - 3\tan x = 4$ are acceptable for A1
M1:	For a correct method of solving their 3TQ in $\tan x$
A1:	Any one of $-\frac{\pi}{4}$, awrt -0.785, awrt 1.326, $-45°$, awrt $75.964°$
A1:	Only $x = -\frac{\pi}{4}, 1.326$ **cao** stated in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$
Note:	**<u>Alternative Method (Alt 1)</u>**

$(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$ **or** $(\sec^2 x - 5)(1 - \cos 2x) = 3\tan x(1 - \cos 2x)$			
Deduces $x = 0$		B1	2.2a
$\sec^2 x - 5 = 3\tan x \Rightarrow \frac{1}{\cos^2 x} - 5 = 3\left(\frac{\sin x}{\cos x}\right)$ $1 - 5\cos^2 x = 3\sin x\cos x$ $1 - 5\left(\frac{1 + \cos 2x}{2}\right) = \frac{3}{2}\sin 2x$	Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A \pm B\cos 2x = \pm C\sin 2x$	M1	2.1
$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ $\{3\sin 2x + 5\cos 2x = -3\}$	$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ o.e.	A1	1.1b
$\sqrt{34}\sin(2x + 1.03) = -3$	Expresses their answer in the form $R\sin(2x + \alpha) = k;\ k \neq 0$ with values for R and α	M1	1.1b
$\sin(2x + 1.03) = -\frac{3}{\sqrt{34}}$			
$x = -\frac{\pi}{4}, 1.326$		A1	1.1b
		A1	1.1b