7. A bacterial culture has area $p \text{ mm}^2$ at time t hours after the culture was placed onto a circular dish. A scientist states that at time t hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture. (a) Show that the scientist's model for p leads to the equation $p = ae^{kt}$ where a and k are constants. **(4)** The scientist measures the values for p at regular intervals during the first 24 hours after the culture was placed onto the dish. She plots a graph of $\ln p$ against t and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95 (b) Estimate, to 2 significant figures, the value of a and the value of k. (3) (c) Hence show that the model for p can be rewritten as $p = ab^t$ stating, to 3 significant figures, the value of the constant b. **(2)** With reference to this model, (d) (i) interpret the value of the constant a, (ii) interpret the value of the constant b. **(2)** (e) State a long term limitation of the model for p. (1)