Question	Scheme	Marks	AOs
5(a)	$f(x)=\dfrac{2x-3}{x^2+4} \Rightarrow f'(x)=\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)^2}$ **or** $f(x)=(2x-3)(x^2+4)^{-1} \Rightarrow f'(x)=2(x^2+4)^{-1}-2x(2x-3)(x^2+4)^{-2}$	M1 A1	1.1b 1.1b
	$f'(x)=\dfrac{-2x^2+6x+8}{(x^2+4)^2}$	A1	1.1b
		(3)	
(b)	$-2x^2+6x+8=0 \Rightarrow -2(x+1)(x-4)=0 \Rightarrow x=-1,\ 4$	B1ft (M1 on EPEN)	1.1b
	Chooses correct region for their numerator and their critical values $x<-1$ or $x>4$	M1 A1	1.1b 2.2a
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts the quotient rule to obtain an expression of the form $\dfrac{P(x^2+4)-Qx(2x-3)}{(x^2+4)^2}$ $P,Q>0$ condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$).

Condone, e.g. $\{f'(x)=\}\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)}$ provided an incorrect formula is not quoted.

May also see the product rule applied to $(2x-3)(x^2+4)^{-1}$ to obtain an expression of the form $\{f'(x)=\}P(x^2+4)^{-1}-Qx(2x-3)(x^2+4)^{-2}$ $P,Q>0$ condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$)

A1: Fully correct differentiation in any form with correct bracketing which may be implied by subsequent working.

A1: $f'(x)=\dfrac{-2x^2+6x+8}{(x^2+4)^2}$ or simplified equivalent, e.g. numerator terms in a different order.

Allow recovery from "invisible" brackets earlier and apply isw once a correct answer is seen. Note that the complete form of the answer is not given so allow candidates to go from e.g.

$f'(x)=\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)^2} \to \dfrac{-2x^2+6x+8}{(x^2+4)^2}$ for the final mark. The denominator $(x^2+4)^2$ may go "missing" on an intermediate line provided it is present in their initial derivative **and** recovered in the final answer. Allow recovery from incorrect expansion of the denominator.

The $f'(x)=$ must appear at some point but allow e.g. "$\dfrac{dy}{dx}=$"

Note that just e.g. $f'(x)=\dfrac{-2(x^2-3x-4)}{(x^2+4)^2}$ without sight of a correct derivative in the correct form scores A0.

(b) **Note:** it is possible to score B0M1A1 in this question due to the demand to "use algebra".
Note: if their numerator from (a) is not a 3 term quadratic then no marks can be scored in (b).

B1ft: Uses algebra to solve their $ax^2+bx+c \ldots 0$ with $a,b,c \neq 0$ where … is any equality or inequality, finding the correct, real critical values for their 3TQ.
The … 0 may be implied by their method.
They must show their working for this mark, so expect to see factorisation, substitution into the correct quadratic formula or completing the square.
Correct values for their quadratic do **not** imply this mark.
Approaches via factorisation must have completely correct factorisation, e.g.

$-2x^2+6x+8\{=0\} \Rightarrow -2(x+1)(x-4)\{=0\} \Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\} \Rightarrow (2x+2)(4-x)\{=0\} \Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\} \Rightarrow x^2-3x-4\{=0\} \Rightarrow (x+1)(x-4)\{=0\} \Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\} \Rightarrow (2x+2)(x-4)\{=0\} \Rightarrow x=-1,\ 4$ scores B0ft

$-2x^2+6x+8\{=0\} \Rightarrow (x+1)(x-4)\{=0\} \Rightarrow x=-1,\ 4$ scores B0ft

M1: Selects the "correct" region for their critical values and their *a* from part (a). Must be *x* not f(*x*)
CVs may have been found using a calculator and may be implied if they are correct for their 3TQ. CVs may be incorrect due to errors in their calculations (but not errors in their method).

- For $a<0$ and roots $\alpha<\beta$ they need e.g. $x<\alpha$, $x>\beta$ (or e.g. $x \leqslant \alpha$ or $x \geqslant \beta$)
- For $a>0$ and roots $\alpha<\beta$ they need e.g. $\alpha<x<\beta$ (or e.g. $x \geqslant \alpha,\ x \leqslant \beta$)

Do not be overly concerned about their use of = , > , < in reference to their $-2x^2+6x+8 \ldots 0$ for this mark or for the A1.
Indicating the region on a sketch is not sufficient. Allow , / or / and / $\cup$ / $\cap$ for the M1.
If they have complex roots (or they use the discriminant to find there are no real roots) then they can score this mark for concluding:

- if *a* < 0, "<u>all values for *x*</u> (have f decreasing)" or "f is <u>always decreasing</u>" or <u>$x \in \mathbb{R}$</u>
- if *a* > 0, "<u>no values for *x*</u> (have f decreasing)" or "f is <u>never decreasing</u>"

A1: Correct solution *x* < −1 or *x* > 4 (allow $x \leqslant -1$ or $x \geqslant 4$) coming from the correct numerator.
Do not isw if they go on to select e.g. *x* > 4 or combine incorrectly to 4 < *x* < −1
Allow full marks to be scored in (b) from an incorrect denominator (but it must be positive for all *x*), e.g. from $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{(x+4)^2}$ or $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{4x^2}$ or $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{x^2+16}$

Examples: Just "*x* < −1 or *x* > 4" stated scores B0M1A1

$-2x^2+6x+8\{=0\} \Rightarrow -2(x+1)(x-4)=0 \Rightarrow x<-1, x>4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\} \Rightarrow x^2-3x-4\{=0\} \Rightarrow (x+1)(x-4)=0 \Rightarrow x \leqslant -1,\ x \geqslant 4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\} \Rightarrow (2x+2)(x-4) \Rightarrow x<-1,\ x>4$ scores B0ftM1A1

$-2x^2+6x+8<0 \Rightarrow x^2-3x-4<0 \Rightarrow (x+1)(x-4)<0 \Rightarrow x<-1, x>4$ scores B1ftM1A1 (as this has correct factorisation shown, the region follows from *a* < 0 (M1) and we condone reference to $x^2-3x-4<0$ as part of their working to find critical values (A1).)

Acceptable notation: allow a "," , "or" , "and" or "$\cup$" to link the two regions, which may also be in set notation. e.g. $x<-1$ or $x>4$ **;** $x \leqslant -1,\ x \geqslant 4$ **;** $x<-1$ and $x>4$
$x \leqslant -1 \cup x \geqslant 4$ **;** $\{x : x<-1 \cup x>4\}$ **;** $\{x \in \mathbb{R} : x \leqslant -1\} \cup \{x \in \mathbb{R} : x \geqslant 4\}$ **;**
$x \in (-\infty,-1) \cup (4,\infty)$ **;** $(-\infty,-1] \cup [4,\infty)$ etc.

Do not accept 4 < *x* < −1 or use of the $\cap$ symbol e.g. $(-\infty,-1] \cap [4,\infty)$ for the final mark, but they may be condoned for the M1. Note also that $[-\infty,-1] \cup [4,\infty]$ scores A0.