13. (a) Express $\dfrac{1}{x(100-x)}$ in partial fractions.

(2)

A scientist is studying the spread of a fungus in a field.
The field has an area of $100\,\text{m}^2$

The area of the field affected by the fungus, $x\,\text{m}^2$, is modelled by the differential equation

$$500\frac{\mathrm{d}x}{\mathrm{d}t} = x(100-x)$$

where t days is the time after the start of the study.

Given that at the start of the study $5\,\text{m}^2$ of the field was affected by the fungus,

(b) solve the differential equation to show that

$$x = \frac{A}{1 + Be^{-\frac{1}{5}t}}$$

where A and B are constants to be found.

(6)

(c) Hence find the area of the field affected by the fungus 10 days after the start of the study.

(2)