11.

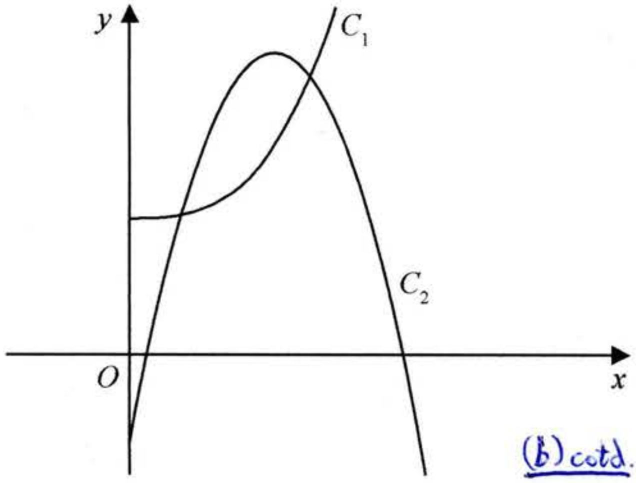

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \frac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

(a) C_1: when $x = \frac{1}{2}$, $y = 2\left(\frac{1}{2}\right)^3 + 10 = \frac{1}{4} + 10 = \frac{41}{4}$

C_2: when $x = \frac{1}{2}$, $y = 42\left(\frac{1}{2}\right) - 15\left(\frac{1}{2}\right)^2 - 7 = 21 - \frac{15}{4} - 7 = \frac{41}{4}$ (1 mark)

so curves intersect at $\left(\frac{1}{2}, \frac{41}{4}\right)$ (1 mark)

(b) for intersection, $2x^3 + 10 = 42x - 15x^2 - 7$

$\Rightarrow 2x^3 + 15x^2 - 42x + 17 = 0$ (1 mark)

we know $x = \frac{1}{2}$ is a solution, so $(2x-1)$ is a factor

$$\begin{array}{r|l} & x^2 + 8x - 17 \\ 2x-1 & 2x^3 + 15x^2 - 42x + 17 \\ & -(2x^3 - x^2) \\ & 16x^2 - 42x \\ & -(16x^2 - 8x) \\ & -34x + 17 \\ & -34x + 17 \\ & 0 \end{array}$$

so, $(2x-1)(x^2+8x-17) = 0$

(2 marks)

(b) cotd.

$x^2 + 8x - 17 = 0$

$\Rightarrow x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-17)}}{2(1)}$

$\Rightarrow x = -4 \pm \sqrt{33}$ (1 mark)

given $x > 0$,

$x = -4 + \sqrt{33}$ (1 mark)