

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \quad 0 \leq t < 25$$

where a and b are constants.

Given that

- the height of the tree was 2.60m, exactly 2 years after being planted
- the height of the tree was 5.10m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures. (4)

Given that the height of the tree was 7m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer. (2)

(a) using given information,

$$\begin{aligned}(2.60)^2 &= a(2) + b \\ (5.10)^2 &= a(10) + b\end{aligned} \quad (1 \text{ mark})$$

$$\Rightarrow \begin{array}{r} 2a + b = 6.76 \\ -10a + b = -26.01 \\ \hline \end{array} \quad (1 \text{ mark})$$

$$\begin{array}{r} -(2a + b = 6.76) \\ \hline 8a = 19.25 \end{array}$$

$$a = 2.40625$$

$$2(2.40625) + b = 6.76 \Rightarrow b = 1.9475 \quad (1 \text{ mark})$$

so,

$$h^2 = 2.41t + 1.95 \quad \text{with } a, b \text{ to 3sf} \quad (1 \text{ mark})$$

(b) Model predicts

$$h^2 = 2.41(20) + 1.95 = 50.15 \quad (1 \text{ mark})$$

$$h = \sqrt{50.15} = 7.081... = 7.08 \text{ m } 3\text{sf}$$

7.08m is close to actual value of 7m

so model is good (1 mark)