Question	Scheme	Marks	AOs
12	$\displaystyle\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4}\times\frac{1}{x}\,dx$	M1	1.1b
	$\displaystyle = \frac{x^4}{4}\ln x - \frac{x^4}{16}(+c)$	M1 A1	1.1b 1.1b
	$\displaystyle\int_1^{e^2} x^3 \ln x \, dx = \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2} = \left(\frac{e^8}{4}\ln e^2 - \frac{e^8}{16}\right) - \left(-\frac{1^4}{16}\right)$	M1	2.1
	$\displaystyle = \frac{7}{16}e^8 + \frac{1}{16}$	A1	1.1b
		(5)	
			(5 marks)

Notes:

M1: Integrates by parts the right way round.

Look for $kx^4 \ln x - \displaystyle\int kx^4 \times \frac{1}{x}\,dx$ o.e. with $k>0$. Condone a missing dx

M1: Uses a correct method to integrate an expression of the form $\displaystyle\int kx^4 \times \frac{1}{x}\,dx \rightarrow c\,x^4$

A1: $\displaystyle\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \frac{x^4}{16}$ $(+c)$ which may be left unsimplified

M1: Attempts to substitute 1 and e^2 into an expression of the form $\pm px^4 \ln x \pm qx^4$, subtracts and uses $\ln e^2 = 2$ (which may be implied).

A1: $\dfrac{7}{16}e^8 + \dfrac{1}{16}$ o.e. Allow $0.4375\,e^8 + 0.0625$ or uncancelled fractions. NOT ISW: $7e^8 +1$ is A0

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You may see attempts where substitution has been attempted.

E.g. $u = \ln x \Rightarrow x = e^u$ and $\dfrac{dx}{du} = e^u$

M1: Attempts to integrate the correct way around condoning slips on the coefficients

$$\int x^3 \ln x \, dx = \int e^{4u}u \, du = \frac{e^{4u}}{4}u - \int \frac{e^{4u}}{4}\,du$$

M1 A1: $\displaystyle\int x^3 \ln x \, dx = \frac{e^{4u}}{4}u - \frac{e^{4u}}{16}(+c)$

M1 A1: Substitutes 0 and 2 into an expression of the form $\pm pue^{4u} \pm qe^{4u}$ and subtracts

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It is possible to use integration by parts "the other way around"

To do this, candidates need to know or use $\displaystyle\int \ln x \, dx = x\ln x - x$

FYI $I = \displaystyle\int x^3 \ln x \, dx = x^3\left(x\ln x - x\right) - \int \left(x\ln x - x\right)\times 3x^2 \, dx = x^3\left(x\ln x - x\right) - 3I + \frac{3}{4}x^4$

Hence $4I = x^4 \ln x - \dfrac{1}{4}x^4 \Rightarrow I = \dfrac{1}{4}x^4 \ln x - \dfrac{1}{16}x^4$

Score M1 for a full attempt at line 1 (condoning bracketing and coefficient slips) followed by M 1 for line 2 where terms in I o.e. to form the answer.