Question	Scheme	Marks	AOs
13	$y^2 - x^2 = 8 \Rightarrow 2y\frac{dy}{dx} - 2x = 0$	M1	2.1
	$\frac{dy}{dx} = \frac{x}{y} \Rightarrow \frac{d^2y}{dx^2} = \frac{y - x\frac{dy}{dx}}{y^2}$	M1 A1	3.1a 1.1b
	$\frac{d^2y}{dx^2} = \frac{y - x\frac{dy}{dx}}{y^2} \Rightarrow y^3\frac{d^2y}{dx^2} = y^2 - xy\frac{dy}{dx} = y^2 - x^2$	M1	3.1a
	$\Rightarrow y^3\frac{d^2y}{dx^2} = y^2 - x^2 = 8 \Rightarrow \frac{d^2y}{dx^2} = \frac{8}{y^3}$ *	A1*	2.1
		(5)	
	Alternative 1:		
	$y^2 - x^2 = 8 \Rightarrow 2y\frac{dy}{dx} - 2x = 0$	M1	2.1
	$2y\frac{dy}{dx} - 2x = 0 \Rightarrow \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} - 1 = 0$	M1 A1	3.1a 1.1b
	$\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} - 1 = 0 \Rightarrow y^3\frac{d^2y}{dx^2} = y^2 - y^2\left(\frac{dy}{dx}\right)^2 = y^2 - x^2$	M1	3.1a
	$\Rightarrow y^3\frac{d^2y}{dx^2} = y^2 - x^2 = 8 \Rightarrow \frac{d^2y}{dx^2} = \frac{8}{y^3}$ *	A1*	2.1
		(5)	
	Alternative 2:		
	$y^2 - x^2 = 8 \Rightarrow 2y\frac{dy}{dx} - 2x = 0$ or $y = \sqrt{x^2+8} \Rightarrow \frac{dy}{dx} = x\left(x^2+8\right)^{-\frac{1}{2}}$	M1	2.1
	$\frac{dy}{dx} = \frac{x}{y} = \frac{x}{\sqrt{x^2+8}} \Rightarrow \frac{d^2y}{dx^2} = \frac{\sqrt{x^2+8} - x^2\left(x^2+8\right)^{-\frac{1}{2}}}{x^2+8}$	M1 A1	3.1a 1.1b
	$\frac{d^2y}{dx^2} = \frac{x^2+8-x^2}{\left(x^2+8\right)^{\frac{3}{2}}}$	M1	3.1a
	$\frac{d^2y}{dx^2} = \frac{8}{y^3}$ *	A1*	2.1
		(5)	
			(5 marks)

Notes
M1: Adopts a correct strategy of implicit differentiation to obtain $\alpha y\frac{\mathrm{d}y}{\mathrm{d}x}-\beta x=0$ M1: Rearranges and then applies the quotient rule to obtain $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\alpha y-\beta x\frac{\mathrm{d}y}{\mathrm{d}x}}{y^2}$ A1: Fully correct differentiation involving the second derivative M1: A complete strategy using the given equation and the first derivative to express the second derivative as an expression not involving the first derivative A1*: Correct proof with no errors **Alternative 1:** M1: Adopts a correct strategy of implicit differentiation to obtain $\alpha y\frac{\mathrm{d}y}{\mathrm{d}x}-\beta x=0$ M1: Differentiates implicitly again using the product rule to obtain $\alpha\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\beta y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+k=0$ A1: Fully correct differentiation involving the second derivative M1: A complete strategy using the given equation and the first derivative to express the second derivative as an expression not involving the first derivative A1*: Correct proof with no errors **Alternative 2:** M1: Adopts a correct strategy of implicit differentiation to obtain $\alpha y\frac{\mathrm{d}y}{\mathrm{d}x}-\beta x=0$ or expresses y explicitly in terms of x and applies the chain rule M1: Differentiates again using the quotient rule A1: Fully correct differentiation M1: Multiplies numerator and denominator by $\left(x^2+8\right)^{\frac{1}{2}}$ A1*: Correct proof with no errors