

Question	Scheme	Marks	AOs
12(a)	$\int x \sin kx \, dx = -\frac{1}{k} x \cos kx + \frac{1}{k} \int \cos kx \, dx$	M1 A1	2.1 1.1b
	$\frac{1}{k^2} \sin kx - \frac{1}{k} x \cos kx + c$	A1*	1.1b
		(3)	
(b)	$\frac{dH}{dt} = \frac{t \sin\left(\frac{\pi t}{5}\right)}{10H} \Rightarrow \int 10H \, dH = \int t \sin\left(\frac{\pi t}{5}\right) dt$	M1	3.1a
	$5H^2 = \frac{25}{\pi^2} \sin\left(\frac{\pi t}{5}\right) - \frac{5}{\pi} t \cos\left(\frac{\pi t}{5}\right) + c$	M1 A1	1.1b 1.1b
	$t = 0, H = 5 \Rightarrow c = 125$	M1	3.4
	$H = \sqrt{\frac{5}{\pi^2} \sin\left(\frac{52\pi}{5}\right) - \frac{1}{\pi} \times 52 \cos\left(\frac{52\pi}{5}\right) + 25}$	M1	1.1b
	$H = 4.51 \text{ m}$	A1	3.2a
		(6)	

(9 marks)

Notes

(a)

M1: Attempts integration by parts and obtains $\pm Ax \cos kx \pm B \int \cos kx \, dx$

A1: For $\int x \sin kx \, dx = -\frac{1}{k} x \cos kx + \frac{1}{k} \int \cos kx \, dx$

A1*: Correct proof

(b)

M1: Attempts to separate the variables to obtain $\int 10H \, dH = \int t \sin\left(\frac{\pi t}{5}\right) dt$ or equivalent e.g.

$$\int H \, dH = \int \frac{1}{10} t \sin\left(\frac{\pi t}{5}\right) dt$$

M1: For using part (a) (or starting again) and integrating both sides to obtain

$$AH^2 = B \sin\left(\frac{\pi t}{5}\right) - Ct \cos\left(\frac{\pi t}{5}\right) (+c) \text{ with or without the “+ c”}$$

$$\text{A1: } 5H^2 = \frac{25}{\pi^2} \sin\left(\frac{\pi t}{5}\right) - \frac{5}{\pi} t \cos\left(\frac{\pi t}{5}\right) (+c) \text{ or equivalent with or without the “+ c”}$$

M1: Substitutes $t = 0$ and $H = 5$ in order to find the constant of integration.

M1: Uses $t = 52$ to find H

A1: Correct height of awrt 4.51 m including units.