

15. (i) Use proof by exhaustion to show that for $n \in \mathbb{N}$, $n \leq 4$

$$(n+1)^3 > 3^n$$

(ii) Given that $m^3 + 5$ is odd, use proof by contradiction to show, using algebra, that m is even.

(4)

(i) By Exhaustion,

	<u>$(n+1)^3$</u>	<u>3^n</u>	
$n=1$	$(1+1)^3 = 8$	$3^1 = 3$	$8 > 3$
$n=2$	$(2+1)^3 = 27$	$3^2 = 9$	$27 > 9$
$n=3$	$(3+1)^3 = 64$	$3^3 = 27$	$64 > 27$
$n=4$	$(4+1)^3 = 125$	$3^4 = 81$	$125 > 81$

(1 mark)

so, for $n \in \mathbb{N}$, $n \leq 4$, $(n+1)^3 > 3^n$
(natural numbers) (1 mark)

(ii) Given $m^3 + 5$ is odd, $m^3 + 5 = 2p + 1$ for some integer p

Assume m is also odd, so $m = 2k + 1$ for some integer k (1 mark)

$$\begin{aligned}
 m^3 + 5 &= (2k+1)^3 + 5 && (1 \text{ mark}) \\
 &= 8k^3 + 12k^2 + 6k + 1 + 5 \\
 &= 8k^3 + 12k^2 + 6k + 6 \\
 &= 2(4k^3 + 6k^2 + 3k + 3) \\
 &= 2q, \text{ where } q \text{ is an integer} \\
 &= \text{even no.} && (1 \text{ mark})
 \end{aligned}$$

$$\begin{aligned}
 &= (2k+1)(2k+1)^2 \\
 &= (2k+1)(4k^2 + 4k + 1) \\
 &= 8k^3 + 8k^2 + 2k + 4k^2 + 4k + 1 \\
 &= 8k^3 + 12k^2 + 6k + 1
 \end{aligned}$$

there is a contradiction here, because $2q \neq 2p + 1$
so if $m^3 + 5$ is odd then m must be even. (1 mark)