

14. Given that

$$y = \frac{x-4}{2+\sqrt{x}} \quad x > 0$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{x}} \quad x > 0$$

where  $A$  is a constant to be found.

(4)

By Quotient Rule,  
if  $y = \frac{u}{v}$ ,

$$y' = \frac{vu' - uv'}{v^2}$$

$$u = x - 4$$

$$u' = 1$$

$$v = 2 + x^{\frac{1}{2}}$$

$$v' = \frac{1}{2} x^{-\frac{1}{2}}$$

$$y' = \frac{(2+x^{\frac{1}{2}})1 - (x-4)\frac{1}{2}x^{-\frac{1}{2}}}{(2+x^{\frac{1}{2}})^2}$$

(2 marks)

$$= \frac{2 + x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}}{(2+x^{\frac{1}{2}})^2}$$

$$= \frac{2 + \frac{1}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}}{(2+x^{\frac{1}{2}})^2}$$

$$\times \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} \quad (1 \text{ mark})$$

$$= \frac{2x^{\frac{1}{2}} + \frac{1}{2}x + 2}{x^{\frac{1}{2}}(2+x^{\frac{1}{2}})^2}$$

$$= \frac{\frac{1}{2}x + 2x^{\frac{1}{2}} + 2}{x^{\frac{1}{2}}(2+x^{\frac{1}{2}})^2} \times \frac{2}{2}$$

$$= \frac{x + 4x^{\frac{1}{2}} + 4}{2x^{\frac{1}{2}}(2+x^{\frac{1}{2}})^2}$$

$$= \frac{(2+x^{\frac{1}{2}})^2}{2x^{\frac{1}{2}}(2+x^{\frac{1}{2}})^2}$$

$$= \frac{1}{2\sqrt{x}}$$

(1 mark)