

13. A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \quad y = \frac{4t}{t^2 + 1} \quad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x-3)^2 + y^2 = 4$$

(3)

$$(x-3)^2 + y^2$$

$$= \left(\frac{t^2 + 5}{t^2 + 1} - 3 \right)^2 + \left(\frac{4t}{t^2 + 1} \right)^2 \quad (1 \text{ mark})$$

$$= \left(\frac{t^2 + 5 - 3(t^2 + 1)}{t^2 + 1} \right)^2 + \left(\frac{4t}{t^2 + 1} \right)^2$$

$$= \left(\frac{-2t^2 + 2}{t^2 + 1} \right)^2 + \left(\frac{4t}{t^2 + 1} \right)^2$$

$$= \frac{4t^4 - 8t^2 + 4}{(t^2 + 1)^2} + \frac{16t^2}{(t^2 + 1)^2}$$

$$= \frac{4t^4 + 8t^2 + 4}{(t^2 + 1)^2} \quad (1 \text{ mark})$$

$$= \frac{4(t^4 + 2t^2 + 1)}{(t^2 + 1)^2} = \frac{4(t+1)^2}{(t+1)^2} = 4 \quad (1 \text{ mark})$$