

11.

(a) strip (trapezium) width,
 $h = \frac{4-2}{4} = \frac{1}{2}$ (1 mark)

From Formula Booklet,

$$\int_2^4 y \, dx$$

$$\approx \frac{1}{2} \left(\frac{1}{2} \right) [0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)]$$

(1 mark)

$$= 2.4085 \dots$$

$$= 2.41 \text{ 3sf (1 mark)}$$

$$\begin{aligned} \text{(b) cota} &= 4(2\ln 2)^2 - 2(\ln 2)^2 - 8(2\ln 2) + 4\ln 2 + 4 \\ &= 4(2)^2(\ln 2)^2 - 2(\ln 2)^2 - 16\ln 2 + 4\ln 2 + 4 \\ &= 16(\ln 2)^2 - 2(\ln 2)^2 - 12\ln 2 + 4 \\ &= 14(\ln 2)^2 - 12\ln 2 + 4 \end{aligned}$$

(1 mark)

$$\begin{aligned} \text{(b) cota} &= 4(\ln 4)^2 - 2(\ln 2)^2 \\ &\quad - 8\ln 4 + 4\ln 2 + 4 - 4 \text{ (1 mark)} \\ &= 4(\ln 2^2)^2 - 2(\ln 2)^2 - 8\ln 2^2 + 4\ln 2 + 4 \end{aligned}$$

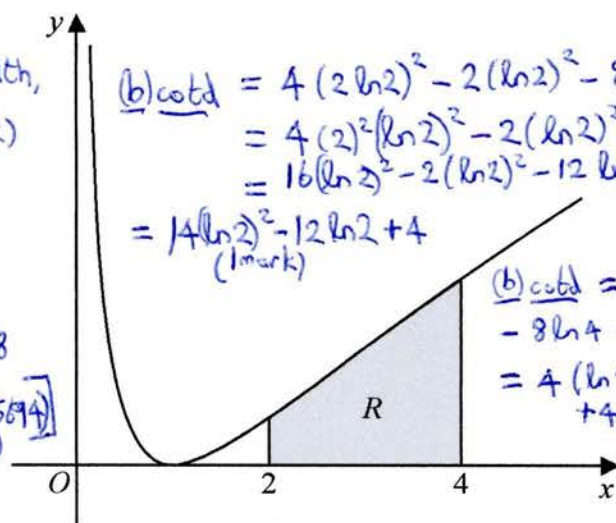


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \quad x > 0$$

The finite region R , shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x -axis and the line with equation $x = 4$

The table below shows corresponding values of x and y , with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R , giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of R , giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where a , b and c are integers to be found.

(b) cota Using Integration by Parts a second time,
 $2\int_2^4 \ln x \, dx = 2\int_2^4 1 \times \ln x \, dx$ (5)

$$\text{(b) cota} \int_2^4 (\ln x)^2 \, dx = [uv]_2^4 - \int_2^4 v u' \, dx$$

$$= [x(\ln x)^2]_2^4 - \int_2^4 x \left(\frac{2\ln x}{x} \right) dx \text{ (2 marks)}$$

$$= [x(\ln x)^2]_2^4 - 2 \left([x \ln x]_2^4 - \int_2^4 x \left(\frac{1}{x} \right) dx \right)$$

$$= [x(\ln x)^2]_2^4 - 2[x \ln x]_2^4 + 2[x]_2^4 \text{ (1 mark)}$$

$$\begin{aligned} u &= \ln x & u' &= \frac{1}{x} \\ v' &= 1 & v &= x \end{aligned}$$