

10.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

FINEVIEW

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan \theta$$

(4)

(b) Hence solve, for $0 < x < 180^\circ$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3 \sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

(a) 2θ on LHS and θ on RHS, so need to reduce 2θ 's using Double Angle Formulae

$$\text{LHS} = \frac{1 - (1 - 2\sin^2 \theta) + 2\sin \theta \cos \theta}{1 + (2\cos^2 \theta - 1) + 2\sin \theta \cos \theta} \quad (2 \text{ marks})$$

$$= \frac{2\sin^2 \theta + 2\sin \theta \cos \theta}{2\cos^2 \theta + 2\sin \theta \cos \theta}$$

$$= \frac{2\sin \theta (\sin \theta + \cos \theta)}{2\cos \theta (\cos \theta + \sin \theta)} = \frac{\sin \theta}{\cos \theta} = \tan \theta = \text{RHS} \quad (2 \text{ marks})$$

There is a choice of Double Angle Formulae for $\cos 2\theta$. The Formulae have been chosen so as to cancel the '1's.

$$(b) \text{ From (a), } \frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = \tan 2x = 3 \sin 2x \quad (1 \text{ mark})$$

$$\frac{\sin 2x}{\cos 2x} = 3 \sin 2x \Rightarrow \sin 2x = 3 \sin 2x \cos 2x \Rightarrow \sin 2x - 3 \sin 2x \cos 2x = 0$$

$$\Rightarrow \sin 2x (1 - 3 \cos 2x) = 0 \Rightarrow \sin 2x = 0 \text{ and } \cos 2x = \frac{1}{3} \quad (1 \text{ mark})$$

$$0 < x < 180^\circ \quad \text{in range } 0^\circ < 2x < 360^\circ, \sin 2x = 0 \Rightarrow 2x = 180^\circ \Rightarrow 0 < 2x < 360^\circ \Rightarrow x = 90^\circ$$

$$\text{in range } 0^\circ < 2x < 360^\circ, \cos 2x = \frac{1}{3} \Rightarrow 2x = 70.528^\circ, 360^\circ - 70.528^\circ = 70.528^\circ, 289.471^\circ$$

$$x = \frac{70.528^\circ}{2}, \frac{289.471^\circ}{2} = 35.26^\circ, 144.73^\circ = 35.3^\circ, 144.7^\circ \text{ 1dp} \quad (2 \text{ marks})$$