

8. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N , in the **first** population is modelled by the equation

$$N = Ae^{kt} \quad t \geq 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M , in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \quad t \geq 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T .

(3)

(a) Given $N = 1000$ when $t = 0$, $1000 = Ae^{k(0)} = Ae^0 = A$ (1 mark)

Given $N = 2 \times 1000$ when $t = 5$,

$$2000 = 1000e^{5k}$$

(1 mark)

$$\Rightarrow e^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = \frac{1}{5} \ln 2$$

(1 mark)

$$\Rightarrow N = 1000e^{(\frac{1}{5} \ln 2)t}$$

(1 mark)

(b) $\frac{dN}{dt} = 1000e^{(\frac{1}{5} \ln 2)t} \times \frac{1}{5} \ln 2$ by Chain Rule (1 mark)

$$= (200 \ln 2)e^{(\frac{1}{5} \ln 2)t} \quad \text{When } t = 8, \frac{dN}{dt} = (200 \ln 2)e^{\frac{8}{5} \ln 2}$$

$$= 420.24 \dots$$

$$= 420 \text{ 2sf bacteria per hr}$$

(1 mark)

$$\text{(c)} \quad 1000e^{(\frac{1}{5} \ln 2)T} = 500e^{1.4(\frac{1}{5} \ln 2)T} \quad (1 \text{ mark})$$

$$2e^{(\frac{1}{5} \ln 2)T} = e^{(0.28 \ln 2)T}$$

$$e^{\ln 2} e^{(0.2 \ln 2)T} = e^{(0.28 \ln 2)T}$$

$$e^{\ln 2 + (0.2 \ln 2)T} = e^{(0.28 \ln 2)T} \quad (1 \text{ mark})$$

$$\Rightarrow \ln 2 + (0.2 \ln 2)T = (0.28 \ln 2)T$$

$$\Rightarrow T = \frac{\ln 2}{0.28 \ln 2 - 0.2 \ln 2}$$

$$= \frac{\ln 2}{0.08 \ln 2}$$

$$= \frac{1}{0.8} = 12.5 \text{ hours} \quad (1 \text{ mark})$$