

7. The circle  $C$  has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

- (i) the coordinates of the centre of  $C$ ,
- (ii) the exact radius of  $C$ , giving your answer as a simplified surd.

(4)

The line  $l$  has equation  $y = 3x + k$  where  $k$  is a constant.

Given that  $l$  is a tangent to  $C$ ,

- (b) find the possible values of  $k$ , giving your answers as simplified surds.

(5)

$$\text{(a)(i)} \quad \underbrace{x^2 - 10x + 25}_{(x-5)^2} - 25 + \underbrace{y^2 + 4y + 4}_{(y+2)^2} - 4 + 11 = 0$$

$$(x-5)^2 - 25 + (y+2)^2 - 4 + 11 = 0$$

$$(x-5)^2 + (y+2)^2 = 18 \quad (2 \text{ marks})$$

$$\text{Centre } C: (5, -2) \quad (1 \text{ mark}) \quad \text{(a)(ii) Radius } C = \sqrt{18} = 3\sqrt{2} \quad (1 \text{ mark})$$

(b) If  $l$  is a tangent, then  $l$  intersects  $C$  at one location  
substituting for  $y$  in circle equation,

$$x^2 + (3x+k)^2 - 10x + 4(3x+k) + 11 = 0 \quad (1 \text{ mark})$$

$$x^2 + 9x^2 + 6kx + k^2 - 10x + 12x + 4k + 11 = 0$$

$$10x^2 + (6k+2)x + (k^2+4k+11) = 0 \quad (1 \text{ mark})$$

$$\text{for one solution, discriminant } b^2 - 4ac = 0 \quad (1 \text{ mark})$$

$$(6k+2)^2 - 4(10)(k^2+4k+11) = 0$$

$$36k^2 + 24k + 4 - 40k^2 - 160k - 440 = 0$$

$$-4k^2 - 136k - 436 = 0$$

$$k^2 + 34k + 109 = 0$$

(1 mark)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow k = \frac{-34 \pm \sqrt{34^2 - 4(1)(109)}}{2}$$

$$\Rightarrow -17 \pm 6\sqrt{5} \quad (1 \text{ mark})$$