



Figure 1

Figure 1 shows a sketch of triangle ABC .

Given that

$$\vec{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$$

$$\vec{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$$

$$\begin{aligned} \text{(a)} \quad \vec{AC} &= \vec{AB} + \vec{BC} \\ &= \begin{pmatrix} -3 \\ -4 \\ -5 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -3+1 \\ -4+1 \\ -5+4 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} \end{aligned}$$

(2 marks)

(a) find $\vec{AC}$

(2)

(b) show that $\cos ABC = \frac{9}{10}$

(3)

$$\text{(b)} \quad \text{length } AB, |\vec{AB}| = \sqrt{(-3)^2 + (-4)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$$

$$\text{length } BC, |\vec{BC}| = \sqrt{1^2 + 1^2 + 4^2} = \sqrt{18} = 3\sqrt{2} \quad (1 \text{ mark})$$

$$\text{length } AC, |\vec{AC}| = \sqrt{(-2)^2 + (-3)^2 + (-1)^2} = \sqrt{14}$$

↑
from (a)

$$\text{Cosine Rule: } b^2 = a^2 + c^2 - 2ac \cos B$$

$$\Rightarrow AC^2 = BC^2 + AB^2 - 2(BC)(AB) \cos ABC$$

$$\Rightarrow 14 = 18 + 50 - 2\sqrt{18}\sqrt{50} \cos ABC \quad (1 \text{ mark})$$

$$\Rightarrow 2\sqrt{18}\sqrt{50} \cos ABC = 18 + 50 - 14 = 54$$

$$\cos ABC = \frac{54}{2\sqrt{18}\sqrt{50}} = \frac{27}{\sqrt{18 \times 50}}$$

$$= \frac{27}{\sqrt{900}} = \frac{27}{30}$$

$$= \frac{9}{10} \quad (1 \text{ mark})$$