

5.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

FINEVIEW

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

(a) geometric sequence with

$$u_1 = 20\,000, r = 1.08$$

$$u_3 = u_1 r^{3-1}$$

$$= u_1 r^2 = 20\,000 (1.08)^2 = \pounds 23,328 \quad (1 \text{ mark})$$

$$(b) \quad u_1 r^{n-1} = 20\,000 (1.08)^{n-1} > 65\,000 \quad (1 \text{ mark})$$

$$(1.08)^{n-1} > \frac{65\,000}{20\,000}$$

$$(1.08)^{n-1} > 3.25$$

$$\ln(1.08)^{n-1} > \ln(3.25)$$

$$(n-1) \ln(1.08) > \ln(3.25)$$

$$n-1 >$$

$$\frac{\ln(3.25)}{\ln(1.08)}$$

(1 mark)

$\ln a > 0$ for $a > 1$
so dividing by $\ln(1.08)$ does not
reverse inequality

$$n > \frac{\ln(3.25)}{\ln(1.08)} + 1 = 16.314 \dots$$

n has to be an integer, so $n = (\text{Year}) 17 \quad (1 \text{ mark})$

$$(c) \quad S_n = \frac{u_1(r^n - 1)}{r - 1} \Rightarrow S_{20} = \frac{20\,000(1.08^{20} - 1)}{1.08 - 1} \quad (1 \text{ mark})$$

$$= 915\,239.2 \dots = \pounds 915\,000 \text{ to nearest } 1000 \quad (1 \text{ mark})$$