

4. The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that α is a solution of the equation

$$\begin{aligned} \text{(a) } f'(x) &= 2x + \frac{1}{(2x^2 - 4x + 5)} \times (4x - 4) \\ &= 2x + \frac{4x - 4}{2x^2 - 4x + 5} \quad \text{(2 marks)} \end{aligned}$$

by chain Rule

$$2x^3 - 4x^2 + 7x - 2 = 0$$

The iterative formula

$$\text{(a) } \text{cota. } f'(x) = \frac{2x(2x^2 - 4x + 5)}{2x^2 - 4x + 5} + \frac{4x - 4}{2x^2 - 4x + 5} \quad (4)$$

$$x_{n+1} = \frac{1}{7} (2 + 4x_n^2 - 2x_n^3)$$

is used to find an approximate value for α .

Starting with $x_1 = 0.3$

$$\begin{aligned} \text{(a) } \text{cota. } f'(x) &= \frac{4x^3 - 8x^2 + 10x + 4x - 4}{2x^2 - 4x + 5} \\ &= \frac{4x^3 - 8x^2 + 14x - 4}{2x^2 - 4x + 5} \end{aligned}$$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2 $\text{(a) } \text{cota. } f'(x) = 0$ when numerator = 0 (1 mark)

(ii) the value of x_4

$$\begin{aligned} &\Rightarrow 4x^3 - 8x^2 + 14x - 4 = 0 \\ &\Rightarrow 2x^3 - 4x^2 + 7x - 2 = 0 \quad (1 \text{ mark}) \end{aligned} \quad (3)$$

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

$$\text{(b)(i) } x_1 = 0.3$$

$$x_2 = \frac{1}{7} (2 + 4(0.3)^2 - 2(0.3)^3) \quad (1 \text{ mark})$$

$$= 0.32942... = 0.3294 \text{ 4dp} \quad (1 \text{ mark})$$

$$\text{(b)(ii) } x_3 = \left\{ \frac{1}{7} (2 + 4(\text{Ans})^2 - 2(\text{Ans})^3) \right\} = 0.33751...$$

$$x_4 = \left\{ \frac{1}{7} (2 + 4(\text{Ans})^2 - 2(\text{Ans})^3) \right\} = 0.33982... = 0.3398 \text{ 4dp} \quad (1 \text{ mark})$$

(c)

$$\begin{array}{ccccccccc} & | & & | & & | & & | & & | \\ 0.3400 & & 0.3405 & & 0.3410 & & 0.3415 & & 0.3420 \end{array}$$

all values round to
0.341 to 3dp

$$\text{So, } f'(x) = 2x^3 - 4x^2 + 7x - 2$$

$$f'(0.3405) = -0.00130...$$

$$f'(0.3415) = +0.00366...$$

(1 mark)

change of sign & f' is continuous
(without asymptotes) so $f' = 0$
between 0.3405 & 0.3415, so
 $\alpha = 0.341 \text{ 3dp} \quad (1 \text{ mark})$