

3. The sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \quad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0 \quad (3)$$

(b) Find the value of k , giving a reason for your answer. (2)

(c) Find the value of u_3 (1)

(a) $u_1 = 2$ given

$$u_2 = k - \frac{24}{2} = k - 12$$

$$u_3 = k - \frac{24}{k-12} = \frac{k^2 - 12k - 24}{k-12} \quad (1 \text{ mark})$$

$$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + \frac{k^2 - 12k - 24}{(k-12)} = 0 \quad (1 \text{ mark})$$

$$\Rightarrow 2(k-12) + 2(k-12)^2 + k^2 - 12k - 24 = 0$$

$$\Rightarrow 2k - 24 + 2(k^2 - 24k + 144) + k^2 - 12k - 24 = 0$$

$$\Rightarrow 3k^2 - 58k + 240 = 0 \quad (1 \text{ mark})$$

(b) Can solve quadratic using calculator,
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$$\Rightarrow k = \frac{40}{3}, 6 \quad (1 \text{ mark})$$

but $k = 6$ given k is an integer (1 mark)

$$\underline{(c)} \quad u_3 = \frac{6^2 - 12(6) - 24}{6-12} = \frac{36 - 72 - 24}{-6} = 10 \quad (1 \text{ mark})$$