13.
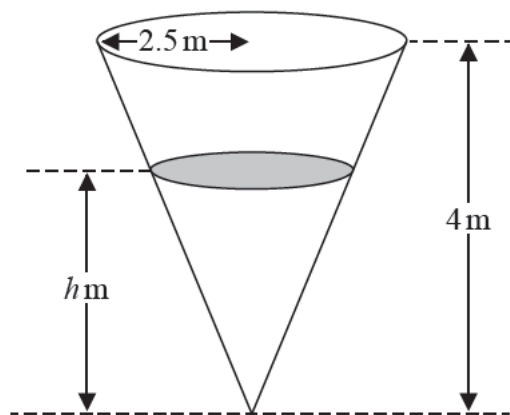

Figure 4

[The volume of a cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$]

Figure 4 shows a container in the shape of an inverted right circular cone which contains some water.

The cone has an internal base radius of $2.5\,\text{m}$ and a vertical height of $4\,\text{m}$.

At time t seconds

- the height of the water is $h\,\text{m}$
- the volume of the water is $V\,\text{m}^3$
- the water is modelled as leaking from a hole at the bottom of the container at a rate of

$$\left(\frac{\pi}{512}\sqrt{h}\right)m^3 s^{-1}$$

(a) Show that, while the water is leaking

$$h^{\frac{3}{2}}\frac{dh}{dt} = -\frac{1}{200}$$

(5)

Given that the container was initially full of water,

(b) find an equation, in terms of h and t, to model this situation.

(3)

It takes approximately 43 minutes for the container to empty.

(c) Use this information to comment on the suitability of this model.

(3)