| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **7** | $\left\{\int xe^{2x}\,dx\right\}$, $\left\{\begin{array}{ll} u = x & \Rightarrow \dfrac{du}{dx} = 1 \\ \dfrac{dv}{dx} = e^{2x} & \Rightarrow v = \dfrac{1}{2}e^{2x} \end{array}\right\}$ | | |
| | $\left\{\int xe^{2x}\,dx\right\} = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\{dx\}$ | M1 | 3.1a |
| | $\left\{\int 2e^{2x} - xe^{2x}\,dx\right\} = e^{2x} - \left(\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\{dx\}\right)$ | M1 | 1.1b |
| | $= e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)$ | A1 | 1.1b |
| | $\text{Area}(R) = \int_0^2 2e^{2x} - xe^{2x}\,dx = \left[\frac{5}{4}e^{2x} - \frac{1}{2}xe^{2x}\right]_0^2$ | M1 | 2.2a |
| | $= \left(\frac{5}{4}e^4 - e^4\right) - \left(\frac{5}{4}e^{2(0)} - \frac{1}{2}(0)e^0\right) = \frac{1}{4}e^4 - \frac{5}{4}$ | A1 | 2.1 |
| | | **(5)** | |
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| **7 Alt 1** | $\left\{\int 2e^{2x} - xe^{2x}\,dx = \int (2-x)e^{2x}\,dx\right\}$, $\left\{\begin{array}{ll} u = 2 - x & \Rightarrow \dfrac{du}{dx} = -1 \\ \dfrac{dv}{dx} = e^{2x} & \Rightarrow v = \dfrac{1}{2}e^{2x} \end{array}\right\}$ | | |
| | $= \frac{1}{2}(2-x)e^{2x} - \int -\frac{1}{2}e^{2x}\{dx\}$ | M1<br>M1 | 3.1a<br>1.1b |
| | $= \frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}$ | A1 | 1.1b |
| | $\left\{\text{Area}(R) = \int_0^2 (2-x)e^{2x}\,dx = \right\}\ \left[\frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}\right]_0^2$ | M1 | 2.2a |
| | $= \left(0 + \frac{1}{4}e^4\right) - \left(\frac{1}{2}(2)e^0 + \frac{1}{4}e^0\right) = \frac{1}{4}e^4 - \frac{5}{4}$ | A1 | 2.1 |
| | | **(5)** | |
| | | | **(5 marks)** |

| **Question 7 Notes:** | |
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| **M1:** | Attempts to solve the problem by recognising the need to apply a method of integration by parts on either $xe^{2x}$ or $(2-x)e^{2x}$. Allow this mark for either<br>• $\pm xe^{2x} \to \pm \lambda xe^{2x} \pm \int \mu e^{2x} \{dx\}$<br>• $(2-x)e^{2x} \to \pm \lambda(2-x)e^{2x} \pm \int \mu e^{2x} \{dx\}$<br>where $\lambda, \mu \neq 0$ are constants. |
| **M1:** | For either<br>• $2e^{2x} - xe^{2x} \to e^{2x} \pm \frac{1}{2}xe^{2x} \pm \int \frac{1}{2}e^{2x} \{dx\}$<br>• $(2-x)e^{2x} \to \pm \frac{1}{2}(2-x)e^{2x} \pm \int \frac{1}{2}e^{2x} \{dx\}$ |
| **A1:** | Correct integration which can be simplified or un-simplified. E.g.<br>• $2e^{2x} - xe^{2x} \to e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)$<br>• $2e^{2x} - xe^{2x} \to e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x}$<br>• $2e^{2x} - xe^{2x} \to \frac{5}{4}e^{2x} - \frac{1}{2}xe^{2x}$<br>• $(2-x)e^{2x} \to \frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}$ |
| **M1:** | Deduces that the upper limit is 2 and uses limits of 2 and 0 on their integrated function |
| **A1:** | Correct proof leading to $pe^4 + q$, where $p = \frac{1}{4}, q = -\frac{5}{4}$ |